Optimal Design of Multistation Assembly Systems

by

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To my family,
Yi, Henglong, Zhifang, Yifei, Kaiping, and Renhua,
who made all of this possible, for their endless encouragement and patience.

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TABLE OF CONTENTS

LIST OF FIGURES

Figure

LIST OF TABLES

Table

LIST OF APPENDICES

Appendix

CHAPTER I

Introduction

1.1 Multistation Assembly Systems

The multistation assembly process is a manufacturing process generally applied when a product cannot be made from a single part due to product functionality, technical infeasibility, or budget limitations.

In general, assembly processes can be classified into two types [Mantripragada 99, Ding 05]. For Type-I assemblies, workpieces are assembled according to their prefabricated mating features. For Type-II assemblies, workpieces are freely positioned by fixtures and there is no interference between workpieces. Figure 1.1 shows examples of Type-I and Type-II assemblies.

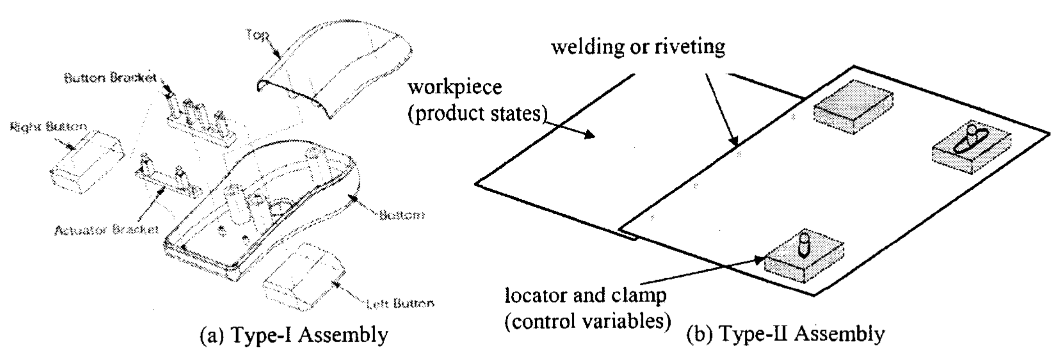

Figure 1.1: Type-I and Type-II assemblies [Ding 05]

For both types of assemblies, variations of manufactured workpieces and manufacturing processes are propagated or accumulated station by station toward a subassembly or a final product. In Type-I assemblies, these accumulated variations cause interference problems, while in Type-II assemblies, the propagated variations may increase or decrease those variations associated with final assembly dimensions. This dissertation focuses on Type-II assembly processes, such as automotive or aircraft body assembly or printed circuit board assembly.

1.1.1 Multistation Assembly System and Its Components

Two or more manufactured workpieces are assembled or joined together using various joining techniques at a station, producing a subassembly or a final product. Figure 1.2 depicts a typical multistation assembly for an automotive body structure. In a body-in-white (BIW) assembly line for the automotive industry, depending on the complexity of the product, there are typically 80 to 130 assembly stations where 150 to 250 sheet metal parts are assembled [Chen 04].

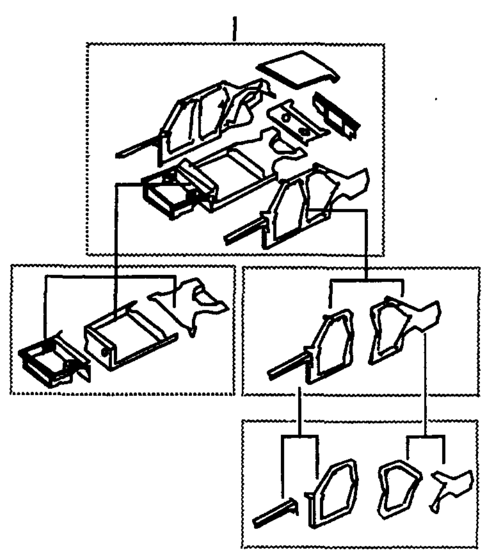

Figure 1.2: Schematic of an automotive body structure assembly [Hu 97]

Physically, a multistation assembly system contains both products and processes. Products include parts, subassemblies, and final products. Processes include fixture elements, assembly tools, and sensor systems. Fixtures are used for locating and holding a part or a subassembly at a three-dimensional workstation. Assembly tools can be welding, riveting, fastening, or other joining techniques. Sensor systems are used to inspect key measurement points along, or at the end of, an assembly process.

Inherently, design evaluation of multistation assembly systems depends on a group of critical features, which are known as key characteristics. Thornton defined key characteristics as follows.

> "A key characteristic is a quantifiable feature of a product or its assemblies, parts, or processes whose expected variation from target has an unacceptable impact on the cost, performance, or safety of the product." [Thornton 03]

In this dissertation, the key features for products are referred to as Key Product Characteristics (KPCs). KPCs are inspected at measurement points to satisfy quality requirements. These measurement points are selected because of their importance to product functionality. The key features for processes are referred to as Key Control Characteristics (KCCs). Early and accurate evaluations of process variations and process configurations are crucial in determining the dimensional accuracy of KPCs; they in turn affect the final dimensional quality of an assembled product.

Both KPCs and KCCs have effects on attributes of multistation assembly systems, i.e., final product quality, process performance, and total cost. The quality of an assembled product, also called dimensional quality, is evaluated using the characteristics of measurement points. Excessive dimensional variations in an automotive body assembly may cause quality issues such as water leakage or wind noise, and process difficulties such as trouble fitting parts together during later operations. Therefore reducing dimensional variations is important for improving the final product quality. The process performance is an evaluation of an assembly process for its ability to improve final product quality and reduce total cost. For example, system robustness describes the sensitivity of dimensional assembly variations with respect to variations of parts, fixtures, and tools. The total cost is all the money spent on products and processes; this would include costs for materials and machines, maintenance costs, quality loss, and costs associated with tolerance specifications of parts, fixtures, and tools.

To summarize, the general objective of multistation assembly system design is to improve final product quality and process performance, while considering the costs for customers and manufacturers.

1.1.2 Multistation Assembly System Models

Manufacturing models have been developed to determine multistation assembly system attributes – final product quality, process performance, and total cost – using engineering and mathematical analysis tools. Decisions can then be made regarding manufacturing process parameters and manufactured part tolerances early in the design and launch stages of manufacturing systems.

Generally, there are three types of manufacturing models: product and process modeling, variation propagation modeling, and cost modeling.

Product and process models are used to describe products, processes, and their interactions in multistation assembly systems. Ceglarek and Shi presented an approach for modeling joint geometry and design gaps [Ceglarek 98]. Ding *et al.* discussed the model of pin-hole contact and they also studied the variation associated with a pin-hole locating pair caused by its clearance [Ding 05]. Liao and Hu presented a technique for modeling fixture-workpiece contact interaction [Liao 98]. Li *et al.* developed a neural network model for on-line nugget size estimation in resistance spot welding based on features extracted from controllable process inputs and on-line signals [Li 00]. Considering dynamic factors like degradation, life-cycle, and maintenance, Ding *et al.* developed a model for the process degradation caused by locator wear processes [Ding 05].

As mentioned earlier, in Type-II assembly, variations in manufactured workpieces and manufacturing processes are propagated station by station toward a subassembly or a final product. The variation propagation processes can be modeled and used to predict variations of final products. Dimensional variation propagation models have been developed for single-station and multistation assembly processes. Station-level models treat the assembly process as if it is conducted in one step. In contrast, multi-station models analyze the process in subsequent steps as the assembly is moved from one station to the next, and include the station-to-station interaction in multistation assembly systems [Mantripragada 99, Jin 99, Lawless 99, Agrawal 99, Suri 99, Djurdjanovic 01, Camelio 03].

Cost modeling serves as an assessment tool for budget estimation. There are several kinds of costs, including: (1) costs for pre-assembly processes, such as costs for set up, machines, tools, materials, and manufactured workpieces; (2) costs for assembly processes, such as costs associated with product and process tolerances, and process configurations; (3) costs for post-assembly processes, such as costs for shipping, replacement, rework, and scrap; (4) costs for labor, such as costs for design, research, planning, operator skills and wages, and inspection. After Taguchi popularized the idea of quality loss [Taguchi 89], cost models were developed using quality loss functions that include the influence of customer satisfaction about products. In order to evaluate the time value of money for quality loss and product degradation, cost models for maintenance and life-cycle design were developed, taking into consideration product volume, production cycle, and market requirements.

1.1.3 Multistation Assembly System Design

The aforementioned manufacturing models enable to design of multistation assembly systems. These models also imply that the design of multistation assembly systems is fundamentally different from that of single-station assembly systems, because the interactions among stations have effects on attributes such as final product quality, process performance, and total cost.

In order to improve final product quality and process performance while reducing total costs, the design of multistation assembly systems has been addressed extensively by researchers who focused on various aspects of the problem. The consideration of numerous design factors and the involvement of various manufacturing models result in complexity and diversity of the design process. A systematic approach is thus necessary to position relevant research works and to locate what has been missing in the literature.

Pahl and Beitz [Pahl 96] presented a systematic approach to machine element embodiment design. Based on this work, Soderberg and Carlson [Soderberg 99] presented an iterative geometry design process for tolerance allocation and robust design, as shown in Figure 1.3. Wickman [Wickman 05] extended the process to include issues related to the evaluation of visual sensitivity and visual quality appearance, and the design process is divided into four main phases: clarification,

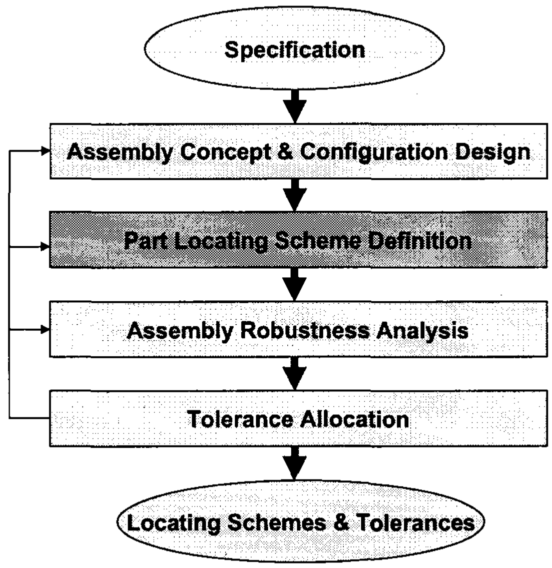

Figure 1.3: Design process for robust assembly design [Soderberg 99]

concept design, embodiment design, and detail design.

In this dissertation, this design classification approach is applied to multistation assembly systems. As shown in Figure 1.4, the research in multistation assembly systems is divided into three main categories: product analysis and configuration design, process analysis and configuration design, and integrated design.

In multistation assembly systems, product analysis and configuration design are conducted to change or adjust characteristics of parts, subassemblies, and final products. The focus is on the total cost and final product quality, noting factors like geometry change, material choices, joint configuration, tolerance analysis, and tolerance allocation. Process analysis and configuration design are conducted to change or adjust characteristics of fixtures, tools, and sensors, with a focus on the process performance and final product quality. This could include assembly sequence design, welding configuration, diagnostic analysis, and fixture layout design. With available manufacturing models, it is possible to conduct several design activities together in

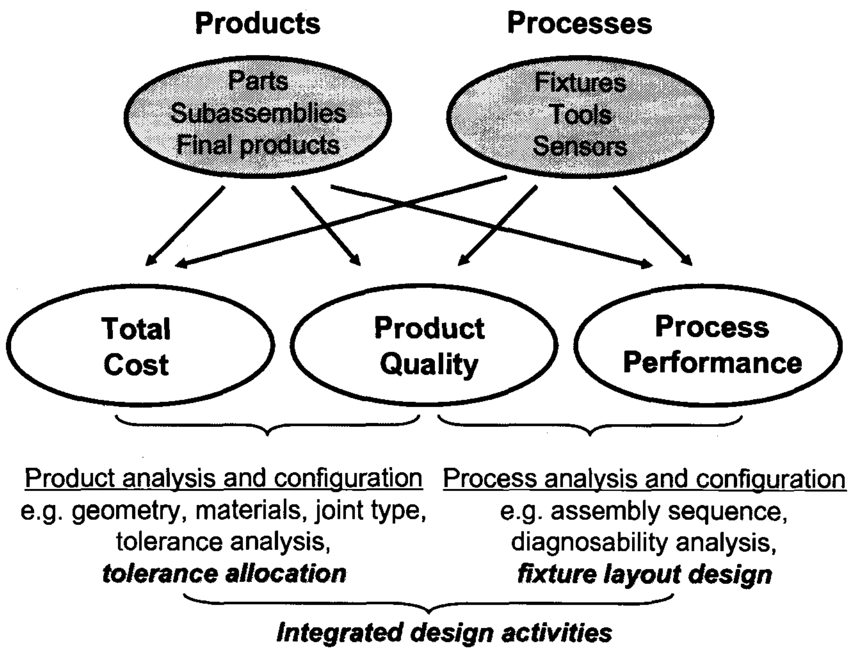

Figure 1.4: Design classification of multistation assembly systems

the early design phase of the assembly system. Integrated design is the task that integrates design activities in the previous two categories. In this dissertation, we focus on tolerance allocation, fixture layout design, and their integrated design.

1.2 Literature Review and Motivation

In this section, the literature in tolerance allocation, fixture layout design, and their integrated design will be reviewed. The limitations of existing methods and applications motivate the research in this dissertation.

1.2.1 Tolerance Allocation

Tolerance is the allowable deviation from a standard, or the range of variation permitted while maintaining a specified dimension. Tolerance allocation is a decision-making process performed early in the product development cycle, before parts are produced and tools are ordered. In this process both product (parts and subassemblies) and process (fixtures and tools) tolerances will be determined with the aid of manufacturing models and design rules in order to maintain final assembly tolerance and cost targets. Tolerance allocation has increasingly gained attention because of the challenges in fast response to product changes, accelerated product development, and increased complexity of products.

Existing Approaches for Tolerance Allocation

There are three key elements to optimally allocate tolerances: tolerance analysis, cost-tolerance relations, and optimization algorithms. After a tolerance definition is presented for tolerance schemes, specifications, and representation, tolerance analysis is used to describe how product and process tolerances transfer to the final product tolerances. From the definition, it can be seen that tolerance synthesis or tolerance allocation is the reverse process of tolerance analysis. Linear tolerance accumulation models, such as worst case tolerance analysis, statistical tolerance analysis, and the Monte Carlo simulation method, are widely used tolerance analysis models. Cost-tolerance relations are cost models associated with both product tolerances and process tolerances. Based on available tolerance analysis models and cost-tolerance relations, appropriate optimization algorithms are then chosen to solve tolerance allocation problems and to evaluate the tradeoff between cost and product quality.

In the current literature, tolerance allocation is conducted by solving a single optimization problem, commonly using an all-in-one (AIO) strategy as follows:

$$\begin{aligned} \min_{\mathbf{t}} \quad & c(\mathbf{t}, \tau) \\ \text{subject to} \quad & \mathbf{g}(\mathbf{t}, \tau) \leq \mathbf{0} \\ & \mathbf{h}(\mathbf{t}, \tau) = \mathbf{0}, \end{aligned} \tag{1.1}$$

where c is the total cost in assembly processes, $\mathbf{t}$ is the product tolerance design vector, and τ is the process tolerance design vector. $\mathbf{g}$ and $\mathbf{h}$ are general inequality and equality constraints, respectively. These constraints represent possible restrictions on final product quality, tolerance limits for product and process design variables, or other operational constraints.

The tolerance allocation optimization problem (1.1) can be linear or nonlinear, with continuous or discrete variables. Accordingly, integer programming, sequential quadratic programming, or simulated annealing methods are typically used to solve the associated optimization problems. For example, Spotts [Spotts 73] developed a nonlinear model that was solved using the method of Lagrange multipliers, to determine assembly tolerances. Lee and Woo [Lee 89] suggested an integer programming model and solved it by the branch and bound method. Chase *et al.* [Chase 90] developed a simple nonlinear model and used exhaustive search, univariate search, sequential quadratic programming, and branch and bound to solve the model. Zhang and Wang [Zhang 93] developed a nonlinear integer model to allocate tolerance with minimum costs. The design problem model was nonconvex with multiple local minima, and was solved using simulated annealing. Chase *et al.*, Chase, Choi *et al.*, and Hong *et al.* provide extensive reviews of existing approaches to solve various formulations [Chase 90, Chase 99a, Chase 99b, Choi 00, Hong 02].

Tolerance Allocation in Assembly Systems

Variation propagation models are important manufacturing models for tolerance allocation in assembly systems. When the relation between tolerance and variation is known, the models can be used to describe how tolerances of manufactured workpieces and manufacturing processes transfer station by station to the tolerances of a subassembly or a final product.

In general, two types of variation propagation models exist, depending on whether the components are considered rigid or compliant. Models for rigid assembly systems consider only in-plane errors. In-plane errors are defined as the errors that occur in the plane normal to the direction of minimum stiffness, while out-of-plane errors can also be defined as the deviations in the most flexible direction of parts. It is assumed that during rigid assembly processes, there is no part deformation in the direction normal to the surface, so that the aggregate behaviors for parts and tooling can be determined by geometric and kinematic relations. Models for compliant assembly systems consider mainly out-of-plane errors. Finite element analysis models are necessary to take into consideration the stiffness of parts and subassemblies, and the forces applied by each tool, thus evaluating the sensitivity matrices in variation propagation models.

Allocation of tolerances has been addressed extensively [Zhang 97], especially for rigid assembly processes. Ding *et al.* [Ding 00b, Ding 01, Ding 05] proposed and demonstrated a framework for process-oriented tolerance synthesis for rigid multistation assembly systems, where process tolerances were optimally allocated by solving a nonlinear constrained optimization problem. The framework was based on the development and integration of three models: tolerance-variation relation, multistation variation propagation, and process degradation. The tolerance-variation model was based on pin-hole fixture mechanisms in multistation assembly processes. Process degradation such as tool wear was also incorporated into the framework, which provided the ability to design tolerances for the whole life-cycle of a production system. The limitation is that only decisions of process tolerances were considered to improve final product quality. The effects of process tolerances on product tolerance allocation schemes are not addressed. Also, manufacturing models and design models for rigid systems might not be applicable for compliant systems because of the involvement of finite element analysis models and the consideration of out-of-plane errors.

Traditional tolerancing methods are not valid in compliant assembly processes [Takezawa 80] and insufficient research has been conducted, even though compliant assemblies are widely used in manufacturing industries such as automotive, aerospace, and electronics. For example, 37-percent of all assembly stations assemble compliant parts in automotive body structure manufacturing [Shiu 97]. Tolerance analysis for compliant processes has been studied [Liu 96, Merkley 98, Sellem 98, Bihlmaier 99]. These research efforts, however, do not consider tolerance allocation.

Shiu *et al.* [Shiu 03] presented a tolerance allocation methodology for compliant beam structures in automotive and aerospace assembly processes. Based on a beam structure model, the method minimized manufacturing costs associated with tolerances of product functional requirements, subject to constraints related to process requirements. Inaccuracy of results due to simplified beam structure models, however, limits further applications, and it is not clear whether this methodology could be extended to compliant multistation assembly systems.

Product and Process Tolerance Allocation for Compliant Multistation Assembly Systems

For the Type-II assemblies addressed in this dissertation, product quality is greatly affected by variations of parts and subassemblies, and by variations of fixtures and tooling elements. The design goal is to minimize costs and final product variations by optimally allocating tolerances of products and processes for compliant multistation assembly systems, using multistation variation propagation models, tolerance-variation (out-of-plane error) models, and cost-tolerance relations.

Additionally, a multilevel optimization strategy will be applied to solve tolerance allocation problems, because tolerance allocation can be modeled as a hierarchical multilevel optimization problem. Specifically, a specific formulation of analytical target cascading (ATC) methodologies [Kim 01] will be used. The ATC structure is applicable to the multistation assembly system design because the structure is close to the tolerance allocation process.

1.2.2 Fixture Layout Design

The dimensional quality of final products depends on both the input variation level and process sensitivity to variation inputs. The former issue can be addressed using tolerance allocation. The latter is handled by determining optimal fixture layouts in a multistation assembly process so that the process is insensitive to variations of incoming parts, fixtures, and tools. Fixture layout design has been addressed extensively in the literature. For the Type-II assemblies, most research focused on "3-2-1" and "N-2-1" fixture locating principles.

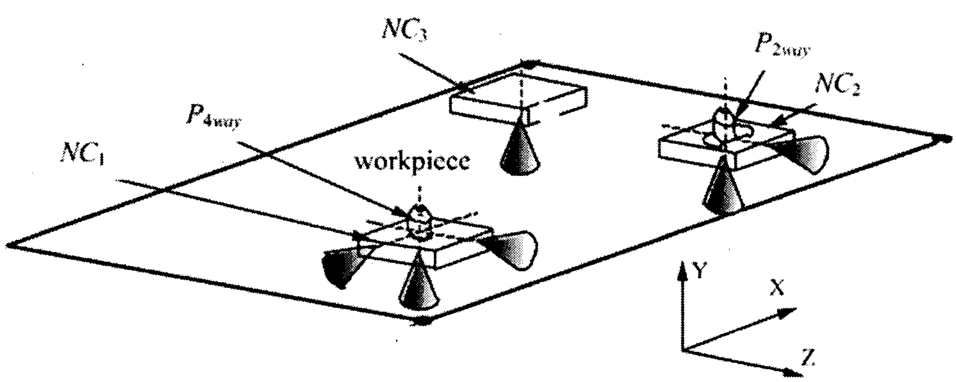

Figure 1.5: Illustration of a "3-2-1" fixture locating principle [Kim 04]

Figure 1.5 illustrates a typical "3-2-1" fixture layout used in this type of assembly processes. A "3-2-1" fixture layout consists of two locating pins, P_{4way} and P_{2way}, and three net contact (NC) blocks (or clamps, or supports), NC_{1-3}. A four-way pin-hole locating pair P_{4way} includes a homogeneous circular hole and pin, and controls part motion in both the X- and Z-directions. A two-way pin-hole locating pair P_{2way} consists of a slot and a circular pin, and thus controls only the motion perpendicular to the long axis of the slot, i.e., the Z-direction. With the two locating pins constraining three degrees of freedom in the X-Z plane, three NC blocks constrain other degrees of freedom of the workpiece. In essence, the "3-2-1" fixture locating principle is used to constrain six degrees of freedom of rigid body motion by implementing a minimum number of features without creating locator interferences. According to the principle, three locators are required for the primary datum surface, two locators for the secondary datum, and one locator for the tertiary datum.

The "N-2-1" fixture locating principle is used when more than three NC blocks $\{NC_k, k = 1, 2, \ldots, N\}$ may be needed in order to reduce the excessive deformation of a workpiece due to gravity and welding forces. As shown in Figure 1.6, it fulfills the fixturing by N ($N \geq 3$), 2, 1, on the primary datum, secondary datum, and tertiary datum, respectively. The additional locators ($N - 3$) are chosen to restrain the deformation in the direction (Y-direction) normal to the surface. The number N is determined by the dimensional specifications of the workpiece. An "N-2-1" fixture layout can be denoted by $\{P_{4way}, P_{2way}, NC_k, k = 1, 2, \ldots, N\}$ in the assembly processes.

Most previous work focused on the "3-2-1" locating principle for both rigid and compliant assemblies, while it has been argued that the "N-2-1" locating principle is better for compliant workpiece fixturing [Cai 96b]. The deformation in the Y-direction cannot be neglected, even under the self-weight of a compliant workpiece. Additional fixtures for compliant assembly processes should be able to restrain ex-

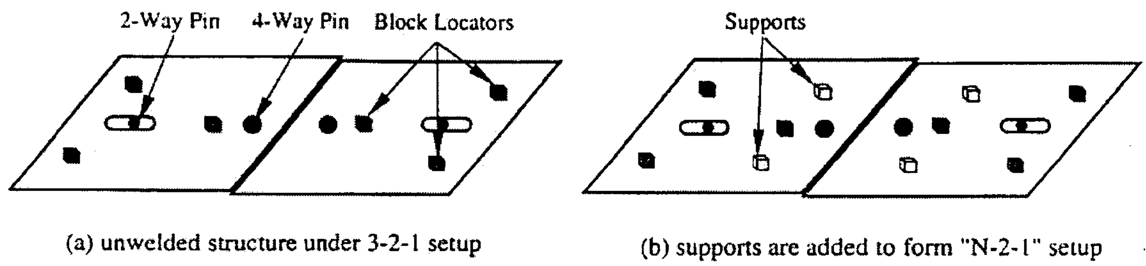

Figure 1.6: "3-2-1" and "N-2-1" fixture locating principles [Cai 96a]

cessive workpiece deformation.

Fixture Layout Design for Rigid Systems

Fixture layout optimization, also called robust fixture layout configuration [Cai 97], aims at improving system robustness by changing the fixture positions, thus altering the sensitivity matrices in variation propagation models. Some research work has been done on fixture layout design of rigid parts.

Ding *et al.* [Ding 02, Ding 01] incorporated process design information, including fixture layouts at individual stations and station-to-station location layout changes. A group of sensitivity indices, which described the system response to variation inputs at both system level and station level, were defined and expressed in terms of the sensitivity matrices in multistation variation propagation models. These models were determined by the given fixture layout configuration.

Kim and Ding [Kim 04] presented a methodology for the optimal design of fixture layouts in multistation assembly processes. Three key aspects of the multistation fixture layout design were addressed: a multistation variation propagation model, a quantitative measure of fixture design, and an effective and efficient optimization algorithm. Based on the rigid multistation variation propagation models, the sensitivity index was defined as the quantitative measure of fixture layout design. Fixture locations were designed to minimize the sensitivity index, thus improving the robustness. The revised exchange algorithm was developed to provide a tradeoff between optimality and efficiency.

Du *et al.* [Du 04] applied this methodology to multistation fixture layout optimization for product variety and concluded that a multistation layout design for a single product is different from a multistation layout design for multiple products. A four-station assembly process for a family of mid-size passenger sedans was used to illustrate relevant concepts and methodologies.

In order to find a competitive design solution with a relatively low computational cost, Kim and Ding [Kim 05] presented a data-mining-aided optimal design method for fixture layout design in rigid multistation assembly systems. The method consisted of four components: a uniform-coverage selection method that chooses design representatives from among a large number of original design alternatives for a nonrectangular design space; feature functions, of which evaluation is computationally economical as the surrogate for the design objective function; a clustering method that generates a design library based on the evaluation of feature functions instead of an objective function; and a classification method to create the design selection rules, eventually leading us to a competitive design.

The above methods, however, are not applicable to compliant multistation assembly systems, because the involvement of finite element analysis (FEA) models results in high computational cost, and requires the integration of optimization algorithms with FEA. Additionally, a sensitivity index needs to be defined based on compliant multistation variation propagation models, in order to evaluate the fixture layout design for compliant multistation assembly systems.

Fixture Layout Design for Compliant Systems

For rigid assembly systems, previous research used dimensions as continuous design variables for fixture locations. Gradient information could be provided for nonlinear programming algorithms. For compliant assembly systems, discrete fixture positions over the part surface are defined using the FEA mesh models, while nonlinear programming algorithms can localize fixtures at any continuous position over the geometric space. This conflict results in optimization difficulties, because a small deviation from the current design position requires a remesh for the finite element models, as pointed out by Rearick *et al.* [Rearick 93]. They proposed an optimization algorithm to determine the optimal number of fixtures and their locations for deformable sheet metal parts. Nonlinear programming was used with finite element analysis for an automobile roof case study. A remeshing algorithm was applied to redefine the nodes and was able to locate the fixtures in the manufacturing model and the optimization algorithm.

The remeshing algorithm makes the computation more complicated. To avoid remeshing finite element models, Cai *et al.* [Cai 96a] proposed the use of multipoint constraint (MPC), one of the advanced features in Nastran [MSC.NASTRAN 01]. MPC was used to calculate sensitivity matrices in variation propagation models. Cai *et al.* also integrated MPC with a gradient-based optimization program to design fixture layouts for two identical flat parts.

Camelio *et al.* [Camelio 04] focused on the impact of fixture positions on the dimensional quality of sheet metal parts after assembly processes in order to understand how the fixture position modifies the contribution of part and tooling errors on the final assembly variations. Specifically, in this work, optimization algorithms integrated with both MPC and remeshing algorithms were necessary to calculate the assembly variation for different fixture positions. When considering part and tooling variation and assembly springback in the process of finite element analysis, it is necessary to apply a unit force over the moving fixture to calculate the fixture sensitivity matrix. MPC cannot be used to obtain fixture sensitivity matrices because it is limited to displacement constraints. Therefore, the model must be remeshed to obtain fixture sensitivity matrices if the effects of fixture variation cannot be neglected.

Additionally, Camelio *et al.* [Camelio 04] used out-of-plane errors in the methodology, while Cai *et al.* [Cai 96b] used in-plane errors. Generally, manufacturing forces are applied in the out-of-plane direction because in-plane forces can produce buckling. For compliant parts, it is preferred that out-of-plane errors be taken into consideration instead of in-plane errors because the deformation in the direction normal to the surface cannot be neglected, even under the weight of the workpiece itself.

Fixture Layout Design for Compliant Multistation Assembly Systems

All aforementioned work was performed to decide the number of fixtures and their locations in rigid assembly systems or compliant single-station assembly systems. No work has been done, however, for fixture layout design in compliant multistation assembly systems.

Fixture layout design for a compliant multistation assembly system is more than the summation of fixture layout designs for the subassembly systems or single compliant parts. The fixture layout designed to minimize component or part deflections at a station may not necessarily lead to a good solution for multistation systems, because it overlooks the effects of variation propagation and the interactions among stations.

One of the objectives of this dissertation is to find the optimal fixture positions, such that the system robustness is maximized for compliant multistation assembly systems. First of all, a sensitivity index needs to be defined in order to evaluate the fixture layout design, taking into consideration assembly variations with respect to variations of parts and fixtures. Because the effects of fixture variation will be considered, both MPC and remeshing algorithms are necessary to calculate the assembly variation for different fixture positions. This results in high computational cost for multistation assembly fixture layout design. It is then required to select appropriate design variables in order to limit the computational cost on finite element models with the Compliant Assembly Variation Analysis (CAVA) software [Hu 00]. Accordingly, appropriate optimization algorithms will be integrated with finite element analysis. In this work, it is assumed that there is no extra cost for setting up or maintaining a robust assembly system. The robustness can be improved only by changing fixture positions.

1.2.3 Integrated Design

Traditionally, tolerance allocation and fixture layout optimization are the two main activities in the design of assembly systems. Tolerance allocation is used to minimize costs and final product variations by optimally allocating tolerances of workpieces and fixtures. Fixture layout optimization serves to improve system robustness by changing the fixture positions, without considering tolerances and costs. With available manufacturing models it is possible to conduct these two design activities together early in the design phase. The task of integrating design activities and considering the relations among total cost, final product quality, and process performance, is referred to as integrated design. In this dissertation, one of the goals is to study integrated design of tolerance allocation and fixture layout design.

In general, successful integrated design is based on the understanding of individual design activities and their interactions. Shared variables and attributes are used to link manufacturing models and design models. Usually, compared to individual design, integrated design includes more design variables and more manufacturing models, which may result in high computational cost and optimization difficulties. Therefore, appropriate optimization strategies are necessary to solve such design problems.

In the current literature there are some integrated design activities for making decisions about product and process characteristics. For example, Zhong *et al.* [Zhong 02a, Zhong 02b] selected process parameters and conducted tolerance allocation studies for machining processes. Chen *et al.* [Jin 01, Chen 01, Chen 04, Chen 05] presented an integrated framework of tolerance and maintenance design for a multistation automotive body assembly process. Optimization problems were formulated to minimize the overall average production cost in the long run, which included costs for tool fabrication, maintenance costs, and the overall loss of quality.

Although these research efforts addressed the integration of dimensional quality and process reliability, the problems were formulated as single-objective (cost-driven) optimization problems. Additionally, the optimization processes for fixture layout design and tolerance allocation are commonly conducted separately. There is no general framework to analyze their interactions qualitatively or quantitatively, or to further study the effects of their integration on total cost, final product quality, and process performance. Therefore, this dissertation was motivated by the need to provide more understanding about the multistation assembly system design by considering tolerance allocation and fixture layout design simultaneously.

As shown in Figure 1.7, the design variables considered in the design problems

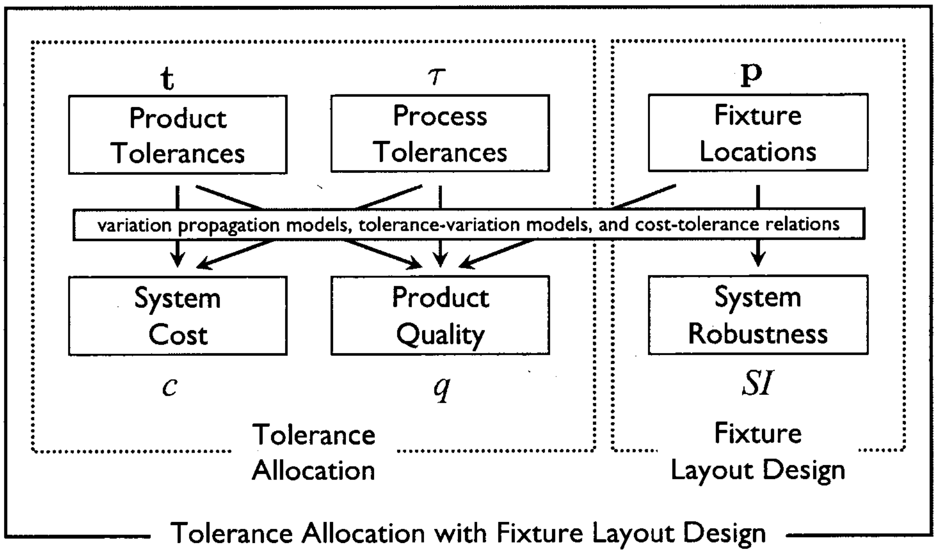

Figure 1.7: Relations among system inputs and major system attributes

are: product tolerances $\mathbf{t}$, process tolerances τ, and fixture locations $\mathbf{p}$. The system cost c depends only on tolerances $\mathbf{t}$ and τ. The sensitivity index SI, an evaluation of system robustness, changes with fixture locations $\mathbf{p}$. Dimensional tolerances, $\mathbf{t}$ and τ, originating from incoming parts and fixture elements on every station, transfer along the production line, to the tolerances of final assembly representing quality q.

The tradeoff between quality and cost is widely known, and it serves as the basis for applications as in tolerance allocation. The system robustness can be improved by conducting fixture layout design. The relations between system cost and system robustness, and the relations between final product quality and system robustness deserve further study. Multiobjective problems will be formulated accordingly. Results will provide more understanding for the decision-making process in multistation assembly system design.

1.3 Objectives

The objective of this dissertation is to contribute to the optimal design of multistation assembly systems. This dissertation aims at answering the following questions.

- How should *tolerance allocation* be conducted for compliant multistation assembly systems?
- How should *fixture layout optimization* be conducted for compliant multistation assembly systems?
- How should design problems be formulated when *tolerance allocation and fixture layout design* are considered simultaneously?
- How should the *appropriate optimization algorithms* be selected to solve these design problems for both rigid and compliant multistation assembly systems?
- How does this work support the decision-making process for the design of multistation assembly systems?

Product and process tolerance allocation will be conducted for compliant multistation assembly systems, using multistation variation propagation models, tolerance-variation models, and cost-tolerance relations. The feasibility of the multilevel optimization strategy will be assessed by applying the analytical target cascading methodology. Specifically, product and process tolerances will be obtained to satisfy overall targets related to available budgets or minimized variations of assembled products.

Optimal fixture layout design for compliant multistation assembly systems will be conducted. A sensitivity index will be defined to evaluate the fixture layout design. Appropriate design variables will be selected to limit the computational costs on finite element models with compliant assembly variation analysis. Accordingly, optimization algorithms will be chosen to be integrated with the finite element analysis.

After tolerance allocation and optimal fixture layout design are conducted independently, this work will demonstrate the effects of design decisions on the assembly system, and provide a basis for advanced studies by considering tolerance allocation and fixture layout design simultaneously. Multiobjective problems will be formulated in order to explore the relations among system cost, final product quality, and system robustness, for both rigid and compliant multistation assembly systems.

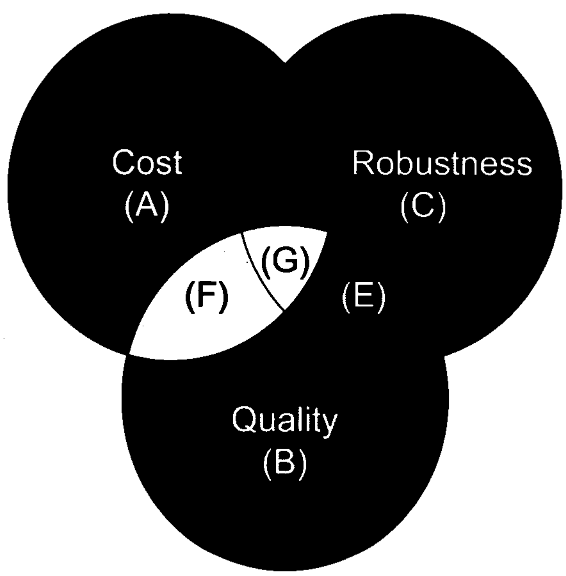

Figure 1.8: Venn diagram for cost, quality, and robustness

The formulation of design problems can be illustrated using a Venn diagram, as shown in Figure 1.8. The circles for cost, quality, and robustness represent design problems A, B, and C, respectively. These three design problems are single objective problems, with the other two attributes as parameters. *Design problem A* is the cost-driven tolerance allocation. The goal is to minimize cost with fixed quality requirements at a certain fixture layout. *Design problem B* is defined as quality-driven tolerance allocation, with the goal of improving final product quality, while satisfying the predefined budgets at a certain fixture layout. *Design problem C* is fixture layout optimization. In this problem, the sensitivity index, an evaluation of system robustness, is determined only by fixture locations; no information about cost or quality is necessary in the design.

The areas where two circles overlap represent two-objective problems, with the remaining attribute as a parameter. *Design problem D* is to study the cost-robustness relations with fixed quality requirements. *Design problem E* is to study the quality-robustness relations with fixed cost requirements. *Design problem F* is to explore the cost-quality tradeoff occurring with a certain fixture layout. This problem can be solved by changing the quality requirement in a cost-driven tolerance allocation problem A, or by changing the predefined budget in a quality-driven tolerance allocation problem B.

The area where three circles overlap represents *design problem G*, which is a three-objective problem examining the relations among cost, quality, and robustness. Table 1.1 lists all the design problems.

Table 1.1: Design problems for multistation assembly systems

Design Problem	Problem Description	Cost	Quality	Robustness
A	Cost-driven Tolerance Allocation	Objective	Parameter	Parameter
B	Quality-driven Tolerance Allocation	Parameter	Objective	Parameter
C	Optimal Fixture Layout Design	N/A	N/A	Objective
D	Cost-robustness Relations with Fixed Quality Requirement	Objective	Parameter	Objective
E	Quality-robustness Relations with Fixed Cost Requirement	Parameter	Objective	Objective
F	Cost-quality Tradeoff with Fixed Fixture Layout	Objective	Objective	Parameter
G	Relations among Cost, Quality, and Robustness	Objective	Objective	Objective

1.4 Outline

The remainder of this dissertation is organized as follows. Chapter II will present the multistation variation propagation models, the tolerance-variation models, and the cost-tolerance relations. All-in-one problems as well as analytical target cascading problems will be formulated. Quality-driven tolerance allocation and cost-driven tolerance allocation will be conducted for the compliant multistation assembly example. Chapter III will demonstrate the optimal fixture layout design for both rigid and compliant multistation assembly systems. In Chapter IV, multiobjective optimization problems will be formulated, and the cost-robustness tradeoff and quality-robustness tradeoff will be explored. Finally, in Chapter V, the dissertation will be summarized, conclusions will be drawn, and suggestions for future work will be given.

CHAPTER II

Product and Process Tolerance Allocation

Tolerance allocation is the decision-making process of determining component and subsystem tolerances with the aid of manufacturing models and design rules in order to reach final assembly quality and budget targets. Traditionally, it is realized by solving a single optimization problem. With increasing requirements for concurrent and consistent design for both products and processes, a methodology is proposed to apply analytical target cascading to the tolerance allocation problem in multistation assembly systems. Specifically, targets of budgets and final product variations are translated into tolerance specifications for incoming parts, subassemblies, and fixtures. Compliant multistation assembly is modeled as a hierarchical manufacturing process and the methodology is demonstrated on an example of vehicle side frame assembly.

2.1 Problem Description

The multistation assembly process of a vehicle side frame structure being considered consists of three stations and four incoming parts, which include a motor rail (Part 1), two door rings (Parts 2 and 3), and a rear quarter (Part 4).

The three-level hierarchy is shown in Figure 2.1. Part 1 and part 2 are assembled at station I to form subassembly 1. Part 3 and part 4 are joined together at station II to form subassembly 2. At station III, the top station, subassembly 1 and 2 are joined to form the final assembly.

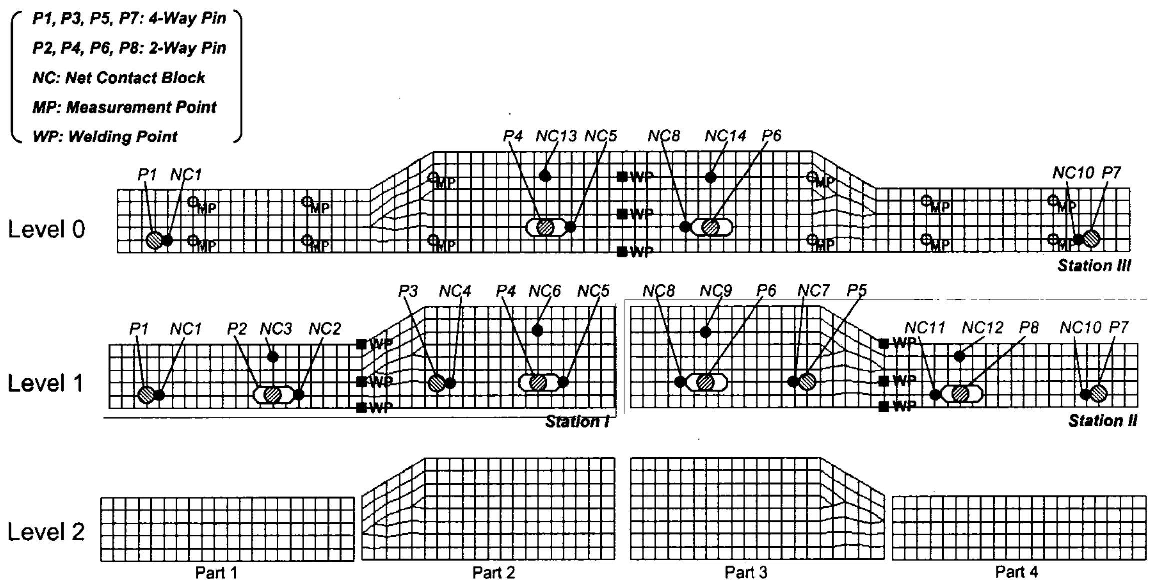

Figure 2.1: Fixture layout in the compliant multistation assembly system

Figure 2.1 also depicts the finite element models. Finite element models are analyzed to take into consideration the stiffness of parts and subassemblies, and the forces applied by each tool, thus evaluating the sensitivity matrices in variation propagation models. A variety of data has been included to completely describe the process on each station. The data includes the material properties, the boundary conditions, and the locations of measurement points, fixture locating points, and welding points. All this information is shared between finite element models and variation propagation models.

In this example, the length and thickness for each incoming part are 700 mm and 2 mm, respectively. The material is mild steel with Young's modulus $E = 2.06$ GPa, and Poisson's ratio $\nu = 0.3$. Locations of measurement points, fixture locating points, and welding point are shown on Figure 2.1.

In this multistation assembly process, the typical "3-2-1" fixture layout is used on every station, consisting of two locating pins, P_{4way} and P_{2way}, and three NC blocks. As a "3-2-1" fixture layout can be denoted by $\{P_{4way}, P_{2way}, NC_k, k = 1, 2, 3\}$, the fixture layout changes in the compliant multistation assembly process can be represented as follows,

$$\begin{aligned}
\text{Station } I &: \{\{P_1, P_2, NC_1, NC_2, NC_3\}, \{P_3, P_4, NC_4, NC_5, NC_6\}\}, \\
\text{Station } II &: \{\{P_5, P_6, NC_7, NC_8, NC_9\}, \{P_7, P_8, NC_{10}, NC_{11}, NC_{12}\}\}, \\
\text{Station } III &: \{\{P_1, P_4, NC_1, NC_5, NC_{13}\}, \{P_6, P_7, NC_8, NC_{10}, NC_{14}\}\}.
\end{aligned}$$

In the following sections, manufacturing models will be reviewed with focus on multistation variation propagation models, tolerance-variation models, and cost-tolerance relations. With the aforementioned information, these models will be applied to this compliant multistation assembly example. Design problems will then be formulated using both all-in-one (AIO) strategy and analytical target cascading (ATC) strategy. By solving these design problems, tolerances of parts, subassemblies, and fixture locations will be allocated accordingly.

2.2 Manufacturing Models

To conduct tolerance allocation for a compliant multistation assembly system, the relations among variation propagation models, tolerance-variation models, and cost-tolerance relations must be addressed. The models to be presented relate to the compliant vehicle example.

2.2.1 Variation Propagation Models

The variation simulation model for a single station level represents the method of influence coefficients proposed by Liu and Hu [Liu 97]. Considering the compliant nature of sheet metal parts, Liu et al. [Liu 96] and Liu and Hu [Liu 97] proposed a model to analyze the effects of component deviations and assembly springback on assembly variation by applying linear mechanics and statistics. The linear model is presented as below,

$$\mathbf{v}_w = \mathbf{S}\mathbf{v}_u, \tag{2.1}$$

where $\mathbf{v}_w$ and $\mathbf{v}_u$ are the dimensional variation vectors of the KPCs of the assembly and its components, respectively. Using finite element methods, they constructed a sensitivity matrix for compliant parts of complex shapes. The sensitivity matrix $\mathbf{S}$ establishes the linear relationship between the incoming part deviation and the output assembly deviation.

In comparison to the station level approach, it is necessary to define an appropriate variation representation in order to track the variation propagation from station to station for a multistation assembly process. The variation simulation process is sequential, i.e., to estimate the variation at station k, it is necessary to know the variation at the previous station $(k-1)$. Moreover, there is a station-to-station interaction introduced by the release of holding fixtures in the current station and the use of new fixtures in subsequent stations.

A multistation assembly process can then be considered as a sequential discrete-time dynamic system, where the time index in the traditional state space model is replaced by a station index. Therefore, a state space representation can be developed to illustrate station-to-station variation propagation in multistation assembly processes [Jin 99].

Based on the station level model (2.1), Camelio *et al.* [Camelio 03] proposed the use of a state space model to represent the dimensional variation propagation in a compliant multistation assembly process. The model identifies three sources of variations in a compliant assembly: component (part) variation, fixture variation, and joining tool (welding gun) variation; and it considers the influence of these variations on the variation of a subassembly at station k,

$$\begin{aligned}\mathbf{x}_k &= \mathbf{A}_k\mathbf{x}_{k-1} + \mathbf{B}_k\mathbf{u}_k + \mathbf{w}_k \\ &= (\mathbf{S}_k - \mathbf{P}_k + \mathbf{I})(\mathbf{I} + \mathbf{M}_k)\mathbf{x}_{k-1} - (\mathbf{S}_k - \mathbf{P}_k + \mathbf{I})\mathbf{M}_k\mathbf{u}_{k1} \\ &\quad -(\mathbf{S}_k - \mathbf{P}_k)(\mathbf{u}_{k2} + \mathbf{u}_{k3}) + \mathbf{w}_k,\end{aligned} \tag{2.2}$$

where $\mathbf{x}_k$ is the dimensional variation state vector that corresponds to dimensional variations of measurement points on the subassembly or the final assembly. The input vectors include the dimensional variation state vectors of the component KPCs $\mathbf{x}_{k-1}$, the "3-2-1" locating fixtures $\mathbf{u}_{k1}$, the $(N-3)$ additional holding fixtures $\mathbf{u}_{k2}$, and the assembly tools (welding gun) $\mathbf{u}_{k3}$. The variation propagation model considers a decomposition of the fixture variation vector into two sets of fixtures: the "3-2-1" locating fixtures and the $(N-3)$ additional holding fixtures. $\mathbf{w}_k$ is the noise vector. If the fixture scheme is "3-2-1" rather than "N-2-1" ($N > 3$) and welding guns are perfect, Equation (2.2) can be simplified as

$$\mathbf{x}_k = (\mathbf{S}_k - \mathbf{P}_k + \mathbf{I})(\mathbf{I} + \mathbf{M}_k)\mathbf{x}_{k-1} - (\mathbf{S}_k - \mathbf{P}_k + \mathbf{I})\mathbf{M}_k\mathbf{u}_{k1} + \mathbf{w}_k. \tag{2.3}$$

In order to obtain the state transition matrix $\mathbf{A}_k$ and the input matrix $\mathbf{B}_k$, their relations to the re-location/re-orientation matrix $\mathbf{M}_k$, the part deformation matrix $\mathbf{P}_k$, the sensitivity matrix $\mathbf{S}_k$, and the noise vector $\mathbf{w}_k$ are derived. $\mathbf{M}_k$ represents how the state vector changes due to the change of the locating scheme from the previous station to the current station. $\mathbf{P}_k$ considers the part deformation during assembly. The method of influence coefficients [Liu 97], with finite element analysis, is applied for each station in order to obtain the deformation matrix $\mathbf{P}_k$ and the sensitivity matrix $\mathbf{S}_k$. The relocation matrix $\mathbf{M}_k$ is defined using homogeneous transformation and the locators' positions. In this work, $\mathbf{w}_k$ is assumed to be zero.

Compliant assembly variation analysis (CAVA) model was developed for obtaining the aforementioned matrices, thus predicting variations in compliant assemblies [Hu 00]. The model relates the dimensional deviations of an assembly to individual component deformations, which can be varied statistically according to variations in non-component factors (machining, fixturing, and tooling). It also considers the springback, the force information, the "N-2-1" locating principle, and additional variations arising from the configuration of the assembly system (series, parallel, or hybrid lines).

The input file of CAVA has the summary of the finite element model information, and the information about measurement points, welding points, fixture locating points, and fixture release points. When applying on compliant assembly processes, CAVA calls Nastran [MSC.NASTRAN 01] to evaluate and analyze the finite element models, and then provides the values of the matrices. By assuming that $\mathbf{w}_k$ is zero, Equation (2.3) can be presented as

$$\mathbf{x}_k = \mathbf{S}_{\mathbf{A}_k}\mathbf{x}_{k-1} + \mathbf{S}_{\mathbf{B}_k}\mathbf{u}_{k1}, \tag{2.4}$$

where $\mathbf{S}_{\mathbf{A}_k}$ and $\mathbf{S}_{\mathbf{B}_k}$ can be obtained using CAVA.

2.2.2 Tolerance-variation Models

Under the assumption of statistically independent variations for components and fixtures, the covariance relationship is derived from Equation (2.4) as

$$\Sigma_{\mathbf{x}_k} = \mathbf{S}_{\mathbf{A}_k}\Sigma_{\mathbf{x}_{k-1}}\mathbf{S}_{\mathbf{A}_k}^T + \mathbf{S}_{\mathbf{B}_k}\Sigma_{\mathbf{u}_{k1}}\mathbf{S}_{\mathbf{B}_k}^T, \text{ and} \tag{2.5}$$

$$\sigma_k^2 = diag(\Sigma_k), \tag{2.6}$$

where the elements of σ_k^2 are the diagonal elements of Σ_k. The vectors $\sigma_{\mathbf{x}_k}^2 = diag(\Sigma_{\mathbf{x}_k})$ and $\sigma_{\mathbf{u}_{k1}}^2 = diag(\Sigma_{\mathbf{u}_{k1}})$ include the KPC variances for the subassembly and the KCC variances for the fixtures, respectively.

Therefore, the tolerance-variation relation for KPCs, KCCs, and their interrelation, can be addressed. Under the assumption that dimensional variations occur randomly, tolerance is related to parameters of probabilistic distributions such as variance or standard deviation. Sometimes additional analysis needs to be conducted, given different contact or location scheme information. In this work, it is assumed that tolerance t is related to standard deviation σ_x according to

$$t = 6\sigma_x. \tag{2.7}$$

The product tolerances $\mathbf{t}$ are the tolerances associated with KPCs. The process tolerances τ are the tolerances associated with KCCs. By applying Equation (2.5), Equation (2.6), and Equation (2.7), the model to describe the transfer of tolerances is derived as,

$$\begin{aligned} \mathbf{t}_k^2 &= \mathbf{S}_{\mathbf{A}_k}^2\mathbf{t}_{k-1}^2 + \mathbf{S}_{\mathbf{B}_k}^2\tau_k^2, \Rightarrow \\ \mathbf{t}_k &= \mathbf{F}_{t_k}(\mathbf{t}_{k-1}, \tau_k) = \sqrt{\mathbf{S}_{\mathbf{A}_k}^2\mathbf{t}_{k-1}^2 + \mathbf{S}_{\mathbf{B}_k}^2\tau_k^2}. \end{aligned} \tag{2.8}$$

For the three-level hierarchical system as shown in Figure 2.1, the tolerance transfer models are presented as below,

$$\begin{aligned}
\mathbf{t}_{01} &= \mathbf{F}_{t_{01}}(\mathbf{t}_{11}, \mathbf{t}_{12}, \tau_{01}) = \sqrt{\mathbf{S}^2_{\mathbf{A}_{01}}[\mathbf{t}_{11}, \mathbf{t}_{12}]^2 + \mathbf{S}^2_{\mathbf{B}_{01}}\tau^2_{01}}, \\
\mathbf{t}_{11} &= \mathbf{F}_{t_{11}}(\mathbf{t}_{21}, \mathbf{t}_{22}, \tau_{11}) = \sqrt{\mathbf{S}^2_{\mathbf{A}_{11}}[\mathbf{t}_{21}, \mathbf{t}_{22}]^2 + \mathbf{S}^2_{\mathbf{B}_{11}}\tau^2_{11}}, \\
\mathbf{t}_{12} &= \mathbf{F}_{t_{12}}(\mathbf{t}_{23}, \mathbf{t}_{24}, \tau_{12}) = \sqrt{\mathbf{S}^2_{\mathbf{A}_{12}}[\mathbf{t}_{23}, \mathbf{t}_{24}]^2 + \mathbf{S}^2_{\mathbf{B}_{12}}\tau^2_{12}},
\end{aligned} \tag{2.9}$$

where $\mathbf{t}_{21}$, $\mathbf{t}_{22}$, $\mathbf{t}_{23}$, and $\mathbf{t}_{24}$ are tolerance vectors for Parts 1, 2, 3, and 4, respectively, at level 2. $\mathbf{t}_{11}$ is the tolerance vector for subassembly 1 at level 1. $\mathbf{t}_{12}$ is the tolerance vector for subassembly 2 at level 1. $\mathbf{t}_{01}$ is the final assembly product tolerance vector at the top station. τ_{01}, τ_{11}, and τ_{12} are process tolerance vectors at stations I, II, III, respectively.

It is known that the tolerance of a clearance is usually larger than 0.01 mm. Thus, in this work, product tolerance is chosen from the interval $[0.01, 2]$ mm, i.e. 0.01 mm $\leq t \leq 2$ mm. For process tolerance, the upper bound is assumed to be one-third of the product tolerance upper bound, so that 0.01 mm $\leq \tau \leq \frac{2}{3}$ mm.

Given tolerance information, the final product quality is represented by

$$q = \|\mathbf{t}_{01}\|_\infty.$$

The use of the infinity norm implies that the quality requirement is imposed on KPCs with relatively large variation values. This representation or evaluation is only one of many possible choices. Other valid measures such as the l_1-norm or the l_2-norm may also be used. It was indicated that the use of the infinity norm is preferred by industrial practitioners [Ding 05].

2.2.3 Cost-tolerance Relations

Using tight tolerances is costly for manufacturers, while loose tolerances may lead to reduced product performance [Hu 92]. It is then necessary to develop cost-tolerance models for tolerance allocation in assembly systems. The tolerance-variation models provide information about product and process tolerances, enabling the evaluation of the system cost through cost-tolerance relations.

Most cost models consider fixed costs and manufacturing costs for each component of the assembly [Kalpakjian 97]. Fixed costs include setting up, fixturing, loading and unloading, handling, tooling, and other pre-processing operations. Manufacturing costs are associated with tolerance specifications of incoming parts, fixtures, and process tools. Manufacturing costs represent the costs of producing a single component dimension to a specified tolerance.

The most widely used cost-tolerance models were developed by Speckhart [Speckhart 72]. He proposed an exponential cost-tolerance model for machining, and suggested an allocation model for both linear and nonlinear design functions using worst-case or statistical approaches. Wu *et al.* [Wu 88] reviewed several cost models and concluded that models that define cost as a combined (exponential/reciprocal power) function of tolerances are the most accurate, followed by models based on exponential relations, and models based on reciprocal relations. Other similar functions can be applied to describe the cost-tolerance relations. A good survey of cost models can be found in [Chase 90]. In early research on tolerance allocation, reciprocal and exponential tolerance-cost models were widely applied [Speckhart 72, Wilde 75, Sutherland 75, Ostwald 77, Wu 88].

Since manufacturing cost is both site- and process-dependent, cost is usually calculated based on empirical relations, using tolerance data of manufacturing processes. If the data is not available, choosing an appropriate cost model depends on a comprehensive understanding of the specific manufacturing system. Reciprocal and exponential cost functions are good alternatives, offering decent data fit and simple function structures. In this work, the exponential function [Speckhart 72] is chosen to represent the cost of the incoming parts of the compliant assembly,

$$c(t) = k_1 + k_2 e^{-k_3 t}, \tag{2.10}$$

where k_1 represents fixed costs, k_2 is the cost of producing a single component dimension to a specified tolerance t, and k_3 describes how sensitive the process cost is to changes in tolerance specifications. For simplicity, these parameters are assumed as $k_1 = 0$, $k_2 = 1$, and $k_3 = 3$, and all process tolerances are subject to the same cost model. For a tolerance vector, the cost-tolerance model is,

$$c(\mathbf{t}) = F_c(\mathbf{t}) = \sum_{i=1}^{n} e^{-3t_i}, \tag{2.11}$$

where n is the size of the tolerance vector.

For the compliant multistation assembly example as shown in Figure 2.1, the system cost c is represented by c_{01}, which is the sum of costs from station I ($c_{11}(\mathbf{t}_{21}, \mathbf{t}_{22}, \tau_{11})$), costs from station II ($c_{12}(\mathbf{t}_{23}, \mathbf{t}_{24}, \tau_{12})$), and costs at station III from process tolerances τ_{01}. The relations are

$$\begin{aligned} c_{01} &= c_{11}(\mathbf{t}_{21}, \mathbf{t}_{22}, \tau_{11}) + c_{12}(\mathbf{t}_{23}, \mathbf{t}_{24}, \tau_{12}) + F_c(\tau_{01}) \\ &= F_c(\mathbf{t}_{21}, \mathbf{t}_{22}, \tau_{11}) + F_c(\mathbf{t}_{23}, \mathbf{t}_{24}, \tau_{12}) + F_c(\tau_{01}), \end{aligned} \tag{2.12}$$

where $\mathbf{t}_{21}$ and $\mathbf{t}_{22}$ are incoming part tolerances at station I, and $\mathbf{t}_{23}$ and $\mathbf{t}_{24}$ are incoming part tolerances at station II. τ_{11}, τ_{12}, and τ_{01} are process tolerances at station I, II, and III, respectively.

2.3 Design Models

Tolerances can be allocated by solving a single optimization problem, i.e., all-in-one (AIO) problem. Analytical target cascading (ATC) strategy can be applied because tolerance allocation can be modeled as a hierarchical multilevel optimization process. Next, the tolerance allocation problem will be formulated using both AIO and ATC strategies.

2.3.1 All-in-one Problem Formulation

The work of tolerance allocation is based on the tradeoffs between system cost and final product quality. A two-objective all-in-one (AIO) optimization problem is formulated as

$$\begin{aligned} \min_{\mathbf{t},\tau} \quad & \{c(\mathbf{t},\tau), q(\mathbf{t},\tau)\} \\ \text{s.t.} \quad & \mathbf{g}(\mathbf{t},\tau) \leq 0, \end{aligned} \tag{2.13}$$

where $\mathbf{t}$ is the product tolerance vector of incoming parts, and τ is the process tolerance vector of fixture locations. c is the system cost, including all costs of incoming parts and costs at stations. q is the final product quality. $\mathbf{g}(\mathbf{t},\tau) \leq 0$ are inequality constraints that represent lower and upper design bounds for $\mathbf{t}$ and τ.

In the general multiobjective problem

$$\begin{aligned} \min_{\mathbf{x}} \quad & \mathbf{f}(\mathbf{x}) \\ \text{s.t.} \quad & \mathbf{g}(\mathbf{x}) \leq 0 \\ & \mathbf{h}(\mathbf{x}) = 0, \end{aligned} \tag{2.14}$$

$\mathbf{f}(\mathbf{x})$ is a vector of objective functions to be minimized with respect to $\mathbf{x}$, $\mathbf{g}(\mathbf{x})$ is a vector of inequality constraint functions, and $\mathbf{h}(\mathbf{x})$ is a vector of equality constraint functions.

One of the strategies to generate the Pareto set is the upper bound method, also referred to as the ε-constraint method. This involves minimizing a primary objective, and expressing the other objectives in the form of inequality constraints:

$$
\begin{aligned}
\min_{\mathbf{x}} \quad & f_k(\mathbf{x}) \\
\text{s.t.} \quad & f_i(\mathbf{x}) \le U_i, i = 1, 2, \ldots, m, \text{and } i \ne k \\
& \mathbf{g}(\mathbf{x}) \le 0 \\
& \mathbf{h}(\mathbf{x}) = 0,
\end{aligned} \tag{2.15}
$$

where U_i is an upper bound, $f_i(\mathbf{x})$ is the ith objective, and m is the total number of objectives. This approach is able to identify solutions on a nonconvex boundary that are not obtainable using the weighted sum technique.

By applying the upper bound method, the Pareto set for Problem (2.13) can be generated by solving a series of cost-driven problems (2.16) with different q_{s_i} or a series of quality-driven problems (2.17) with different b_i as follows,

$$
\begin{aligned}
\min_{\mathbf{t},\tau} \quad & c(\mathbf{t}, \tau) \\
\text{s.t.} \quad & q(\mathbf{t}, \tau) \le q_{s_i} \\
& \mathbf{g}(\mathbf{t}, \tau) \le 0, \ \text{and},
\end{aligned} \tag{2.16}
$$

$$
\begin{aligned}
\min_{\mathbf{t},\tau} \quad & q(\mathbf{t}, \tau) \\
\text{s.t.} \quad & c(\mathbf{t}, \tau) \le b_i \\
& \mathbf{g}(\mathbf{t}, \tau) \le 0,
\end{aligned} \tag{2.17}
$$

where $\mathbf{t}$ is the product tolerance vector of incoming parts, and τ is the process tolerance vector of fixture locations; c is the system cost, including all costs of incoming parts and costs at stations; q is the final product quality; $\mathbf{g}(\mathbf{t}, \tau) \le 0$ are inequality constraints that represent lower and upper design bounds for $\mathbf{t}$ and τ; q_{s_i} is the quality requirement; and b_i is the budget.

This cost-driven problem formulation is exactly the same as a typical problem formulation for minimum cost tolerance allocation. This quality-driven problem formulation can be seen as a problem formulation for variation reduction as studied in the manufacturing literature.

2.3.2 Analytical Target Cascading Problem Formulation

Analytical target cascading (ATC) is a multilevel optimization methodology used at the early design stages of system design [Kim 01]. Exploiting the multilevel hierarchy of a decomposed system, top-level design targets are propagated ("cascaded") to the appropriate subsystem and component design specifications in a consistent and efficient manner.

ATC operates by formulating and solving a minimum target deviation optimization problem for each element of the multilevel hierarchy. Assuming that inputs of higher level elements are the outputs of lower-level elements, it aims at minimizing the gap between what upper-level elements "want" and what lower-level elements "can". The objective is to identify early in the design process the relations and possible tradeoffs among elements, and to determine specifications that yield consistent system design with minimized deviation from design targets.

Theoretical convergence properties of ATC have been proven for appropriate coordination strategies [Michelena 03]. The formulation has been extended to probabilistic formulation for multilevel optimization under uncertainty [Kokkolaras 06, Liu 06]. Weighting update method was proposed for better efficiency and acceptable consistency deviation [Michalek 05b]. Also, the ATC process has been applied successfully to diverse problems in automotive design [Kim 02, Kim 03a, Kim 03b, Kokkolaras 04, Li 04], aircraft design [Allison 06], product family design [Kokkolaras 02], building systems design [Choudhary 03, Choudhary 05b, Choudhary 05a], and linking engineering and business decisions under marketing considerations [Michalek 05a, Cooper 06, Kim 06].

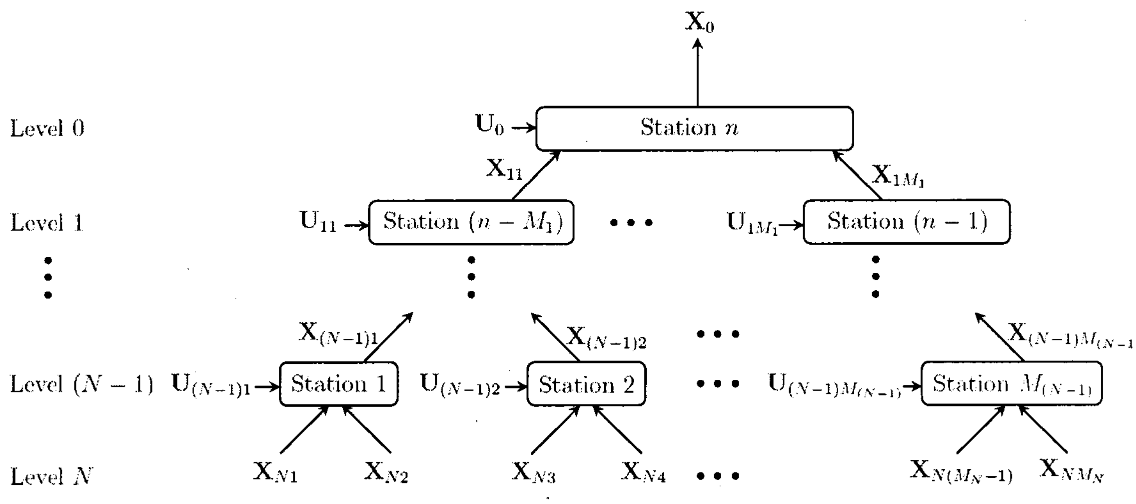

Figure 2.2: Variation models as an ATC process

As shown in Figure 2.2, in a multistation assembly process, variations from incoming parts (at level N) and fixture variations (from level $(N-1)$ to level 0) are propagated level by level to variations of the final product. This multilevel hierarchical structure enables the application of ATC to tolerance allocation. The general mathematical formulation of the ATC process is presented in detail in the aforementioned references. Here, the specific formulation is presented as it applies to the tolerance allocation problem in compliant multistation assembly systems.

Tolerance allocation in multistation assembly systems can be modeled as an ATC process. At the top level (the final station), where all the subassemblies are joined to form the final assembly, target tolerance values are assigned for product and process tolerances of the final assembly. Using variation propagation models, tolerance-variation relations, and cost-tolerance models, these target values can be cascaded level-by-level down through the subassemblies, all the way to the bottom level, and then rebalanced up based on the tolerance specifications of the incoming parts. The cost associated with each station is accounted for during the process.

The ATC process consists of solving a sequence of optimization subproblems. Each subproblem is associated with a single station. The mathematical formulation of the tolerance allocation subproblem at station j of level i is

$$\begin{aligned} \min_{\mathbf{z}_{ij}} \quad & \|\mathbf{t}_{ij} - \mathbf{t}_{ij}^H\|_2^2 + \sum_{k=1}^{M_{ij}} \|\mathbf{t}_{(i+1)k} - \mathbf{t}_{(i+1)k}^L\|_2^2 + (c_{ij} - c_{ij}^H)^2 + \sum_{k=1}^{M_{ij}} (c_{(i+1)k} - c_{(i+1)k}^L)^2 \\ \text{s. t.} \quad & \mathbf{g}_{ij}(\mathbf{t}_{(i+1)1}, \ldots, \mathbf{t}_{(i+1)M_{ij}}, \tau_{ij}, c_{(i+1)1} \ldots, c_{(i+1)M_{ij}}) \leq \mathbf{0} \\ & \mathbf{h}_{ij}(\mathbf{t}_{(i+1)1}, \ldots, \mathbf{t}_{(i+1)M_{ij}}, \tau_{ij}, c_{(i+1)1} \ldots, c_{(i+1)M_{ij}}) = \mathbf{0}, \end{aligned} \tag{2.18}$$

where M_{ij} denotes the number of "child" stations of station j. $\mathbf{t}_{(i+1)1}, \ldots, \mathbf{t}_{(i+1)M_{ij}}$ represent product tolerance vectors of parts or subassemblies at the "child" stations. τ_{ij} is the process tolerance vector of fixture locators at the current station. $c_{(i+1)1}, \ldots, c_{(i+1)M_{ij}}$ are costs accumulated up to the "child" stations. The input design vector $\mathbf{z}_{ij}$ consists of $\mathbf{t}_{(i+1)1}, \ldots, \mathbf{t}_{(i+1)M_{ij}}$, $c_{(i+1)1}, \ldots, c_{(i+1)M_{ij}}$, and τ_{ij}.

What enables the formulation and implementation of the ATC process is that $\mathbf{t}_{ij} = \mathbf{F}_{t_{ij}}(\mathbf{t}_{(i+1)1}, \ldots, \mathbf{t}_{(i+1)M_{ij}}, \tau_{ij})$ and $c_{ij} = F_{c_{ij}}(\mathbf{t}_{(i+1)1}, \ldots, \mathbf{t}_{(i+1)M_{ij}}, \tau_{ij}) + \sum_{k=1}^{M_{ij}} c_{(i+1)k}$, according to Equations (2.8) and (2.11). $\mathbf{t}_{ij}$ is the output product tolerance vector of subassemblies or final assembly. Note that cost is accumulated (as opposed to being propagated) during the assembly process. The total cost c_{ij} of a station j at level i is defined as the sum of costs accumulated up to the "child" stations plus the costs of the station process, which depends on both product and process tolerances.

In the hierarchy, $\mathbf{t}_{ij}^H$ and c_{ij}^H denote quantities computed at higher levels. $\mathbf{t}_{(i+1)1}^L, \ldots,$

$\mathbf{t}^L_{(i+1)M_{ij}}$ and $c^L_{(i+1)1}, \ldots, c^L_{(i+1)M_{ij}}$ denote quantities computed at lower levels.

The inequality and equality constraints ($\mathbf{g}_{ij}$ and $\mathbf{h}_{ij}$, respectively) represent possible restrictions on final product quality, tolerance limits for product and process design variables, or other operational constraints. Budget allocation can also be realized by setting budget constraints. The manufacturing cost allocated for each part or subassembly is required to be less than its budget target.

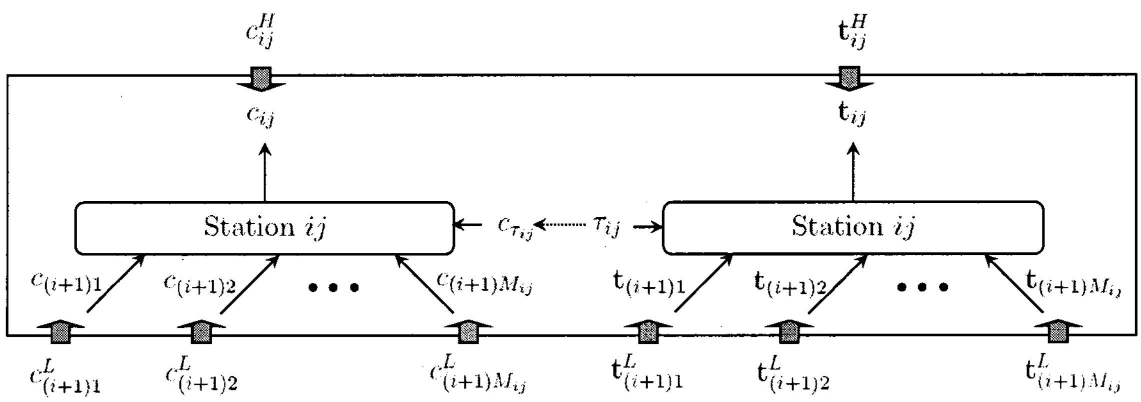

Figure 2.3: Information flow in ATC

As also depicted in Figure 2.3, the subproblem aims at minimizing deviations between current tolerance and cost values and target values cascaded from the higher level. It also enforces consistency by taking into account the tolerances and the costs that can be expected from the "child" stations.

Note that the top-level station does not have a "parent" station. Tolerance and cost targets at top-level station are given by the management. Similarly, bottom-level stations do not have "child" stations. The objective terms related to consistency are not included in the formulation of bottom-level subproblems.

The optimization subproblems are solved using the MATLAB implementation of the sequential quadratic programming (SQP) algorithm [MATLAB 04]. For a two-level problem, the top-level problem is solved first. Targets for the bottom-level problems are obtained and cascaded. The bottom-level problems are then solved independently to match the cascaded targets. Bottom-level tolerances are then passed up to the top-level problem, completing one ATC iteration. This process is repeated until the optimization variable values do not change significantly after successive ATC iterations.

Next, four scenarios will be presented to demonstrate the feasibility of the proposed ATC methodology.

2.4 Quality-driven Product and Process Tolerance Allocation

Product tolerances are allocated for the compliant multistation assembly system example to improve the final product quality. This is known as the quality-driven product tolerance allocation. Two scenarios are presented, with local budget constraints or global budget constraints. For each scenario, problems are formulated and solved using both AIO and ATC strategies.

2.4.1 Tolerance Allocation with Local Budget Constraints

One way to evaluate the cost-quality tradeoff is to constrain the money supply for each station. In this scenario, local budget constraints ensure that money spent to purchase each incoming part is equal and will not exceed the budget b_l. The AIO problem formulation for quality-driven product tolerance allocation with local budget constraint is presented below,

$$\begin{aligned}
\min_{\mathbf{t}_{21},\mathbf{t}_{22},\mathbf{t}_{23},\mathbf{t}_{24}} \quad & \|\mathbf{t}_{01}(\mathbf{t}_{21},\mathbf{t}_{22},\mathbf{t}_{23},\mathbf{t}_{24})\|_\infty \\
\text{s.t.} \quad & c_{21}(\mathbf{t}_{21}) \le b_l \\
& c_{22}(\mathbf{t}_{22}) \le b_l \\
& c_{23}(\mathbf{t}_{23}) \le b_l \\
& c_{24}(\mathbf{t}_{24}) \le b_l \\
& 0.01\text{mm} \le \mathbf{t}_{21},\mathbf{t}_{22},\mathbf{t}_{23},\mathbf{t}_{24} \le 2\text{mm}.
\end{aligned} \tag{2.19}$$

where $\mathbf{t}_{21}$, $\mathbf{t}_{22}$, $\mathbf{t}_{23}$, and $\mathbf{t}_{24}$ are the tolerance vectors of incoming parts, c_{21}, c_{22}, c_{23}, and c_{24} are the costs of incoming parts, and $\mathbf{t}_{01}$ is the tolerance vector of final assembly product. Please refer to Equations (2.9) and (2.11) for tolerance transfer models and cost-tolerance relations.

Product quality targets (final product tolerances $\mathbf{t}_{01}^H$) are assigned by management. In this work, $\mathbf{t}_{01}^H$ is set as $\mathbf{0}$ to be consistent with the AIO formulation. The mathematical formulation of the ATC process is as follows,

(Level 0: only one station)

$$\begin{aligned}
\min_{\mathbf{t}_{11},\mathbf{t}_{12}} \quad & \|\mathbf{t}_{01}(\mathbf{t}_{11},\mathbf{t}_{12}) - \mathbf{t}_{01}^H\|_2^2 + \|\mathbf{t}_{11} - \mathbf{t}_{11}^L\|_2^2 + \|\mathbf{t}_{12} - \mathbf{t}_{12}^L\|_2^2 \\
\text{s.t.} \quad & \mathbf{t}_{11},\mathbf{t}_{12} \ge 0,
\end{aligned}$$

(Level 1, Station 1)

$$\begin{aligned}
\min_{\mathbf{t}_{21},\mathbf{t}_{22}} \quad & \|\mathbf{t}_{11}(\mathbf{t}_{21},\mathbf{t}_{22}) - \mathbf{t}_{11}^H\|_2^2 \\
\text{s.t.} \quad & c_{21}(\mathbf{t}_{21}) \le b_l \\
& c_{22}(\mathbf{t}_{22}) \le b_l \\
& 0.01\text{mm} \le \mathbf{t}_{21},\mathbf{t}_{22} \le 2\text{mm},
\end{aligned}$$

(Level 1, Station 2)

$$
\begin{aligned}
\min_{\mathbf{t}_{23},\mathbf{t}_{24}} \quad & \|\mathbf{t}_{12}(\mathbf{t}_{23},\mathbf{t}_{24}) - \mathbf{t}_{12}^{H}\|_2^2 \\
\text{s.t.} \quad & c_{23}(\mathbf{t}_{23}) \leq b_l \\
& c_{24}(\mathbf{t}_{24}) \leq b_l \\
& 0.01\text{mm} \leq \mathbf{t}_{23}, \mathbf{t}_{24} \leq 2\text{mm}.
\end{aligned}
$$

where $\mathbf{t}_{21}$, $\mathbf{t}_{22}$, $\mathbf{t}_{23}$, and $\mathbf{t}_{24}$ are the tolerance vectors of incoming parts, c_{21}, c_{22}, c_{23}, and c_{24} are the costs of incoming parts, $\mathbf{t}_{11}$ and $\mathbf{t}_{12}$ are the tolerance vector of subassemblies, and $\mathbf{t}_{01}$ is the tolerance vector of final assembly product. Please refer to Equations (2.9) and (2.11) for tolerance transfer models and cost-tolerance relations.

2.4.2 Tolerance Allocation with Global Budget Constraints

In this scenario, the total cost accumulated from bottom levels is required to be less than the global budget b_g, where $b_g = \sum_{i=1}^{m} b_{l_i}$. m is the number of incoming parts at bottom levels, and b_{l_i} is the local budget for ith incoming parts. Local budgets are not necessarily equal. The AIO problem formulation for quality-driven product tolerance allocation with global budget constraint is presented below:

$$
\begin{aligned}
\min_{\mathbf{t}_{21},\mathbf{t}_{22},\mathbf{t}_{23},\mathbf{t}_{24}} \quad & \|\mathbf{t}_{01}(\mathbf{t}_{21},\mathbf{t}_{22},\mathbf{t}_{23},\mathbf{t}_{24})\|_\infty \\
\text{s.t.} \quad & c_{01}(\mathbf{t}_{21},\mathbf{t}_{22},\mathbf{t}_{23},\mathbf{t}_{24}) \leq b_g \\
& 0.01\text{mm} \leq \mathbf{t}_{21},\mathbf{t}_{22},\mathbf{t}_{23},\mathbf{t}_{24} \leq 2\text{mm}.
\end{aligned} \tag{2.20}
$$

where $\mathbf{t}_{21}$, $\mathbf{t}_{22}$, $\mathbf{t}_{23}$, and $\mathbf{t}_{24}$ are the tolerance vectors of incoming parts, $\mathbf{t}_{01}$ is the tolerance vector of final assembly product, and c_{01} is the total system cost of the assembly process. Please refer to Equations (2.9) and (2.11) for tolerance transfer models and cost-tolerance relations.

In ATC, the purchasing costs for incoming parts are treated as design variables and are determined by solving the top level problem. The mathematical formulation of the ATC process is as follows.

(Level 0: only one station)

$$\begin{aligned}
\min_{\mathbf{t}_{11},\mathbf{t}_{12},c_{11},c_{12}} \quad & \|\mathbf{t}_{01}(\mathbf{t}_{11},\mathbf{t}_{12}) - \mathbf{t}_{01}^{H}\|_2^2 + \\
& \|\mathbf{t}_{11} - \mathbf{t}_{11}^{L}\|_2^2 + \|\mathbf{t}_{12} - \mathbf{t}_{12}^{L}\|_2^2 + (c_{11} - c_{11}^{L})^2 + (c_{12} - c_{12}^{L})^2 \\
\text{s.t.} \quad & c_{01}(c_{11},c_{12}) = c_{11} + c_{12} \leq b_g \\
& \mathbf{t}_{11},\mathbf{t}_{12},c_{11},c_{12} \geq 0,
\end{aligned}$$

(Level 1, Station 1)

$$\begin{aligned}
\min_{\mathbf{t}_{21},\mathbf{t}_{22}} \quad & \|\mathbf{t}_{11}(\mathbf{t}_{21},\mathbf{t}_{22}) - \mathbf{t}_{11}^{H}\|_2^2 + (c_{11}(\mathbf{t}_{21},\mathbf{t}_{22}) - c_{11}^{H})^2 \\
\text{s.t.} \quad & 0.01\text{mm} \leq \mathbf{t}_{21},\mathbf{t}_{22} \leq 2\text{mm},
\end{aligned}$$

(Level 1, Station 2)

$$\begin{aligned}
\min_{\mathbf{t}_{23},\mathbf{t}_{24}} \quad & \|\mathbf{t}_{12}(\mathbf{t}_{23},\mathbf{t}_{24}) - \mathbf{t}_{12}^{H}\|_2^2 + (c_{12}(\mathbf{t}_{23},\mathbf{t}_{24}) - c_{12}^{H})^2 \\
\text{s.t.} \quad & 0.01\text{mm} \leq \mathbf{t}_{23},\mathbf{t}_{24} \leq 2\text{mm}.
\end{aligned}$$

where $\mathbf{t}_{21}$, $\mathbf{t}_{22}$, $\mathbf{t}_{23}$, and $\mathbf{t}_{24}$ are the tolerance vectors of incoming parts, $\mathbf{t}_{11}$ and $\mathbf{t}_{12}$ are the tolerance vector of subassemblies, $\mathbf{t}_{01}$ is the tolerance vector of final assembly product, c_{21}, c_{22}, c_{23}, and c_{24} are the costs of incoming parts, c_{11} and c_{12} are the costs accumulated up to the corresponding stations, and c_{01} is the total system cost of the assembly process. Please refer to Equations (2.9) and (2.11) for tolerance transfer models and cost-tolerance relations.

2.4.3 Results

For these two scenarios, a parametric study was conducted for tolerance allocation under different budgets. Since the only cost is incurred from purchasing incoming parts, the idea is to investigate the effects of local budgets for incoming parts on the final product quality. The parametric study allowed us to identify cost-quality tradeoffs quantitatively, i.e., determine how much quality improves with increased budgets of incoming parts.

The percentage improvement of final product quality (variation reduction) is depicted in Figure 2.4. It is observed that budget increases result in improvement of final product quality in both scenarios. The second scenario, however, yields greater variation reduction. This can be explained by noting Figure 2.5, where percentage reduction in variations of incoming parts are depicted.

In the first scenario, the budget is increasing equally for each incoming part, so variation reduction is the same for all parts, according to the cost-tolerance relations and tolerance-variation models. In the second scenario, a global budget constraint must be satisfied. The latter allows flexibility and results in allocating resources to more sensitive parts (in this case parts 3 and 4), which in turn results in better final product quality.

2.5 Cost-driven Product and Process Tolerance Allocation

Product tolerances can be allocated for a compliant multistation assembly system by minimizing the system cost. This is known as cost-driven product and process tolerance allocation. Two scenarios will be presented: with, and without, process tolerance allocation. For these two scenarios, problems are formulated and solved using both AIO and ATC strategies.

2.5.1 Product Tolerance Allocation

This scenario focuses on the total cost spent on purchasing incoming parts. Dimensional variation is affected mainly by the variability of parts, fixtures, and joining

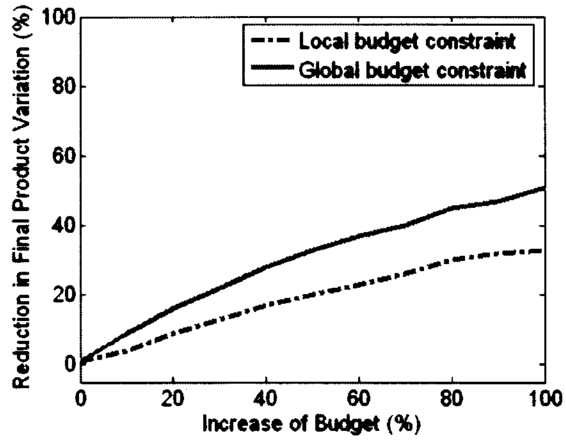

Figure 2.4: Reduction in final product variation as a result of increased budget for both scenarios

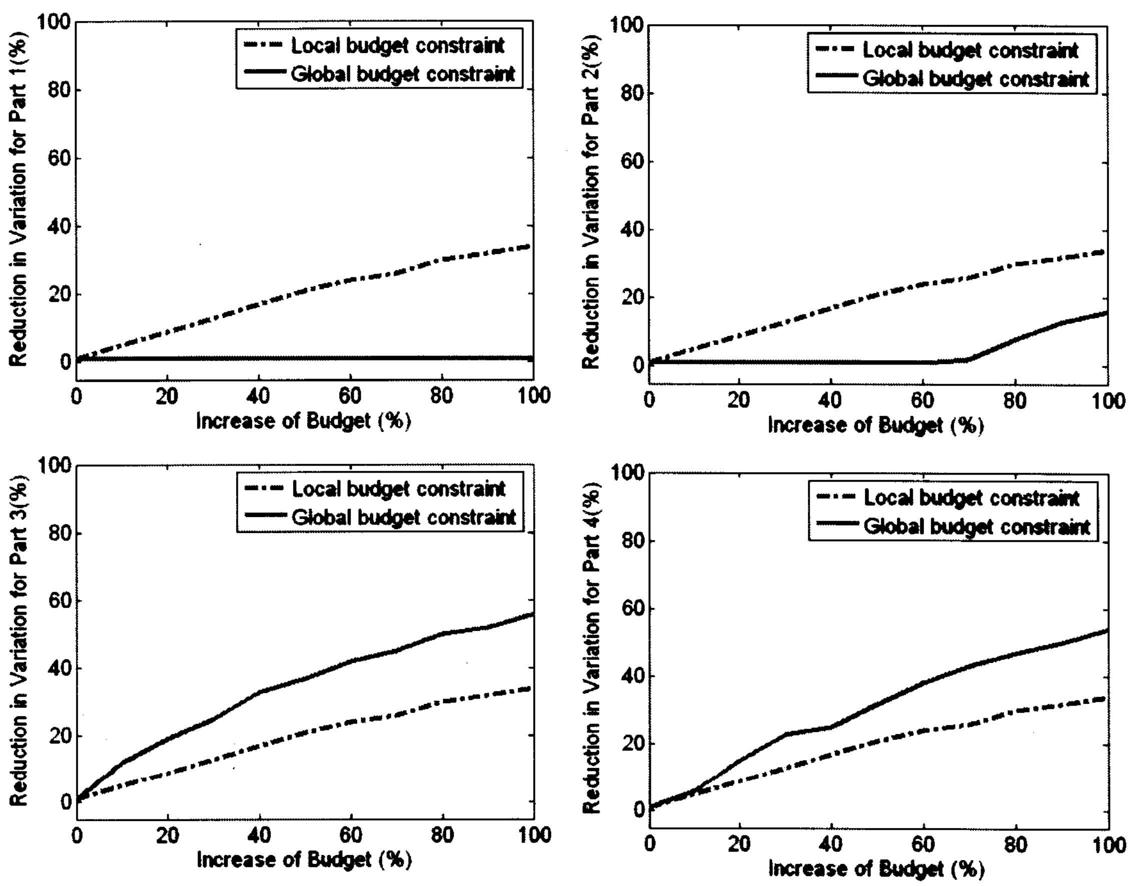

Figure 2.5: Reduction in part product variation as a result of increased budget for both scenarios

methods at each of the multiple stations. As a first step, only the variability of parts is taken into account. The fixture locators are assumed to have no out-of-plane error and are assumed to be positioned at their design-nominal positions.

For both this scenario and the next scenario, tolerances of all final product KPCs must be less than a given upper tolerance limit. In current industrial practice, final products have six-sigma tolerance target value of $q_s = 1.5$ mm. The quality constraint ensures that final product quality will satisfy the quality requirement q_s. The AIO cost-driven problem formulation without process tolerance allocation is presented below:

$$\begin{aligned} \min_{\mathbf{t}_{21},\mathbf{t}_{22},\mathbf{t}_{23},\mathbf{t}_{24}} \quad & c_{01}(\mathbf{t}_{21},\mathbf{t}_{22},\mathbf{t}_{23},\mathbf{t}_{24}) \\ \text{s.t.} \quad & \|\mathbf{t}_{01}(\mathbf{t}_{21},\mathbf{t}_{22},\mathbf{t}_{23},\mathbf{t}_{24})\|_\infty \leq q_s \\ & 0.01\text{mm} \leq \mathbf{t}_{21},\mathbf{t}_{22},\mathbf{t}_{23},\mathbf{t}_{24} \leq 2\text{mm}. \end{aligned} \tag{2.21}$$

where $\mathbf{t}_{21}$, $\mathbf{t}_{22}$, $\mathbf{t}_{23}$, and $\mathbf{t}_{24}$ are the tolerance vectors of incoming parts, $\mathbf{t}_{01}$ is the tolerance vector of final assembly product, and c_{01} is the total system cost of the assembly process. Please refer to Equations (2.9) and (2.11) for tolerance transfer models and cost-tolerance relations.

The cost target c_{01}^H is assigned by management. In this work, c_{01}^H is set as 0, to be consistent with the AIO formulation. The mathematical formulation of the ATC process is as follows:

(Level 0: only one station)

$$\begin{aligned} \min_{\mathbf{t}_{11},\mathbf{t}_{12},c_{11},c_{12}} \quad & (c_{01}(c_{11},c_{12}) - c_{01}^H)^2 + \\ & \|\mathbf{t}_{11} - \mathbf{t}_{11}^L\|_2^2 + \|\mathbf{t}_{12} - \mathbf{t}_{12}^L\|_2^2 + (c_{11} - c_{11}^L)^2 + (c_{12} - c_{12}^L)^2 \\ \text{s.t.} \quad & \|\mathbf{t}_{01}(\mathbf{t}_{11},\mathbf{t}_{12})\|_\infty \leq q_s \\ & \mathbf{t}_{11},\mathbf{t}_{12},c_{11},c_{12} \geq 0, \end{aligned}$$

(Level 1, Station 1)

$$\begin{array}{ll} \min\limits_{\mathbf{t}_{21},\mathbf{t}_{22}} & \|\mathbf{t}_{11}(\mathbf{t}_{21},\mathbf{t}_{22}) - \mathbf{t}_{11}^{H}\|_2^2 + (c_{11}(\mathbf{t}_{21},\mathbf{t}_{22}) - c_{11}^{H})^2 \\ \text{s.t.} & 0.01\text{mm} \leq \mathbf{t}_{21},\mathbf{t}_{22} \leq 2\text{mm}, \end{array}$$

(Level 1, Station 2)

$$\begin{array}{ll} \min\limits_{\mathbf{t}_{23},\mathbf{t}_{24}} & \|\mathbf{t}_{12}(\mathbf{t}_{23},\mathbf{t}_{24}) - \mathbf{t}_{12}^{H}\|_2^2 + (c_{12}(\mathbf{t}_{23},\mathbf{t}_{24}) - c_{12}^{H})^2 \\ \text{s.t.} & 0.01\text{mm} \leq \mathbf{t}_{23},\mathbf{t}_{24} \leq 2\text{mm}. \end{array}$$

where $\mathbf{t}_{21}$, $\mathbf{t}_{22}$, $\mathbf{t}_{23}$, and $\mathbf{t}_{24}$ are the tolerance vectors of incoming parts, $\mathbf{t}_{11}$ and $\mathbf{t}_{12}$ are the tolerance vector of subassemblies, $\mathbf{t}_{01}$ is the tolerance vector of final assembly product, c_{11} and c_{12} are the costs accumulated up to the corresponding stations, and c_{01} is the total system cost of the assembly process. Please refer to Equations (2.9) and (2.11) for tolerance transfer models and cost-tolerance relations.

2.5.2 Product and Process Tolerance Allocation

This scenario considers both product and process tolerance design variables. The changes in tolerance values for fixture locators result in extra costs for the station. Therefore, process tolerances and associated costs must be included in the problem formulation. The AIO cost-driven problem formulation with process tolerance allocation is presented below:

$$\begin{array}{ll} \min\limits_{\mathbf{t}_{21},\mathbf{t}_{22},\mathbf{t}_{23},\mathbf{t}_{24},\tau_{11},\tau_{12},\tau_{01}} & c_{01}(\mathbf{t}_{21},\mathbf{t}_{22},\mathbf{t}_{23},\mathbf{t}_{24},\tau_{11},\tau_{12},\tau_{01}) \\ \text{s.t.} & \|\mathbf{t}_{01}(\mathbf{t}_{21},\mathbf{t}_{22},\mathbf{t}_{23},\mathbf{t}_{24},\tau_{11},\tau_{12},\tau_{01})\|_\infty \leq q_s \\ & 0.01\text{mm} \leq \mathbf{t}_{21},\mathbf{t}_{22},\mathbf{t}_{23},\mathbf{t}_{24} \leq 2\text{mm} \\ & 0.01\text{mm} \leq \tau_{11},\tau_{12},\tau_{01} \leq \frac{2}{3}\text{mm}. \end{array} \tag{2.22}$$

where $\mathbf{t}_{21}$, $\mathbf{t}_{22}$, $\mathbf{t}_{23}$, and $\mathbf{t}_{24}$ are the tolerance vectors of incoming parts, $\mathbf{t}_{01}$ is the tolerance vector of final assembly product, c_{01} is the total system cost of the assembly process, and τ_{01}, τ_{11}, and τ_{12} are process tolerance vectors of fixture locators. Please refer to Equations (2.9) and (2.11) for tolerance transfer models and cost-tolerance relations.

The mathematical formulation of the ATC process is as follows,

(Level 0: only one station)

$$\begin{aligned}
\min_{\mathbf{t}_{11},\mathbf{t}_{12},\tau_{01},c_{11},c_{12}} \quad & (c_{01}(c_{11},c_{12},\tau_{01}) - c_{01}^{H})^2 + \\
& \|\mathbf{t}_{11} - \mathbf{t}_{11}^{L}\|_2^2 + \|\mathbf{t}_{12} - \mathbf{t}_{12}^{L}\|_2^2 + (c_{11} - c_{11}^{L})^2 + (c_{12} - c_{12}^{L})^2 \\
\text{s.t.} \quad & \|\mathbf{t}_{01}(\mathbf{t}_{11},\mathbf{t}_{12},\tau_{01})\|_\infty \le q_s \\
& \mathbf{t}_{11},\mathbf{t}_{12},c_{11},c_{12} \ge 0 \\
& 0.01\text{mm} \le \tau_{01} \le \tfrac{2}{3}\text{mm},
\end{aligned}$$

(Level 1, Station 1)

$$\begin{aligned}
\min_{\mathbf{t}_{21},\mathbf{t}_{22},\tau_{11}} \quad & \|\mathbf{t}_{11}(\mathbf{t}_{21},\mathbf{t}_{22},\tau_{11}) - \mathbf{t}_{11}^{H}\|_2^2 + (c_{11}(\mathbf{t}_{21},\mathbf{t}_{22},\tau_{11}) - c_{11}^{H})^2 \\
\text{s.t.} \quad & 0.01\text{mm} \le \mathbf{t}_{21},\mathbf{t}_{22} \le 2\text{mm} \\
& 0.01\text{mm} \le \tau_{11} \le \tfrac{2}{3}\text{mm},
\end{aligned}$$

(Level 1, Station 2)

$$\begin{aligned}
\min_{\mathbf{t}_{23},\mathbf{t}_{24},\tau_{12}} \quad & \|\mathbf{t}_{12}(\mathbf{t}_{23},\mathbf{t}_{24},\tau_{12}) - \mathbf{t}_{12}^{H}\|_2^2 + (c_{12}(\mathbf{t}_{23},\mathbf{t}_{24},\tau_{12}) - c_{12}^{H})^2 \\
\text{s.t.} \quad & 0.01\text{mm} \le \mathbf{t}_{23},\mathbf{t}_{24} \le 2\text{mm} \\
& 0.01\text{mm} \le \tau_{12} \le \tfrac{2}{3}\text{mm}.
\end{aligned}$$

where $\mathbf{t}_{21}$, $\mathbf{t}_{22}$, $\mathbf{t}_{23}$, and $\mathbf{t}_{24}$ are the tolerance vectors of incoming parts, $\mathbf{t}_{11}$ and $\mathbf{t}_{12}$ are the tolerance vector of subassemblies, $\mathbf{t}_{01}$ is the tolerance vector of final assembly product, c_{11} and c_{12} are the costs accumulated up to the corresponding stations, c_{01} is the total system cost of the assembly process, and τ_{01}, τ_{11}, and τ_{12} are process tolerance vectors of fixture locators. Please refer to Equations (2.9) and (2.11) for tolerance transfer models and cost-tolerance relations.

2.5.3 Results

For these two scenarios, Figure 2.6 compares the results obtained by including and by not including process tolerance allocation in the problem formulation. The quality requirement is satisfied, but most tolerance values of the final assembly increase when considering tolerances of fixture locators. Product tolerances of incoming parts must decrease accordingly in order to satisfy the requirements for final product quality.

While product tolerances of subassembly 1 increase, product tolerances of subassembly 2 decrease. The reason is that parts 3 and 4, and thus subassembly 2, are much more sensitive than parts 1 and 2 (and thus subassembly 1). Within the same cost unit, it is more effective to reduce tolerances of parts 3 and 4 to achieve better final assembly quality. The increased product tolerances of subassembly 1 in scenario 4 are a consequence of both part 1 and part 2 having reached their upper design bounds and the introduced fixture variation.

For product tolerance allocation, only the variability of parts was taken into account. The fixture locators were assumed to have no out-of-plane error and were assumed to be positioned at their design-nominal positions, which is the ideal case. Another parametric study was conducted for different assumed tolerances of fixture locators in cost-driven product tolerance allocation. The results are shown in Table 2.1.

Case 1 is the product tolerance allocation with no error for fixture locators as in Problem (2.21). Case 7 is the product and process tolerance allocation as in Problem (2.22). Cases 1-6 show that the allocated tolerances for incoming parts

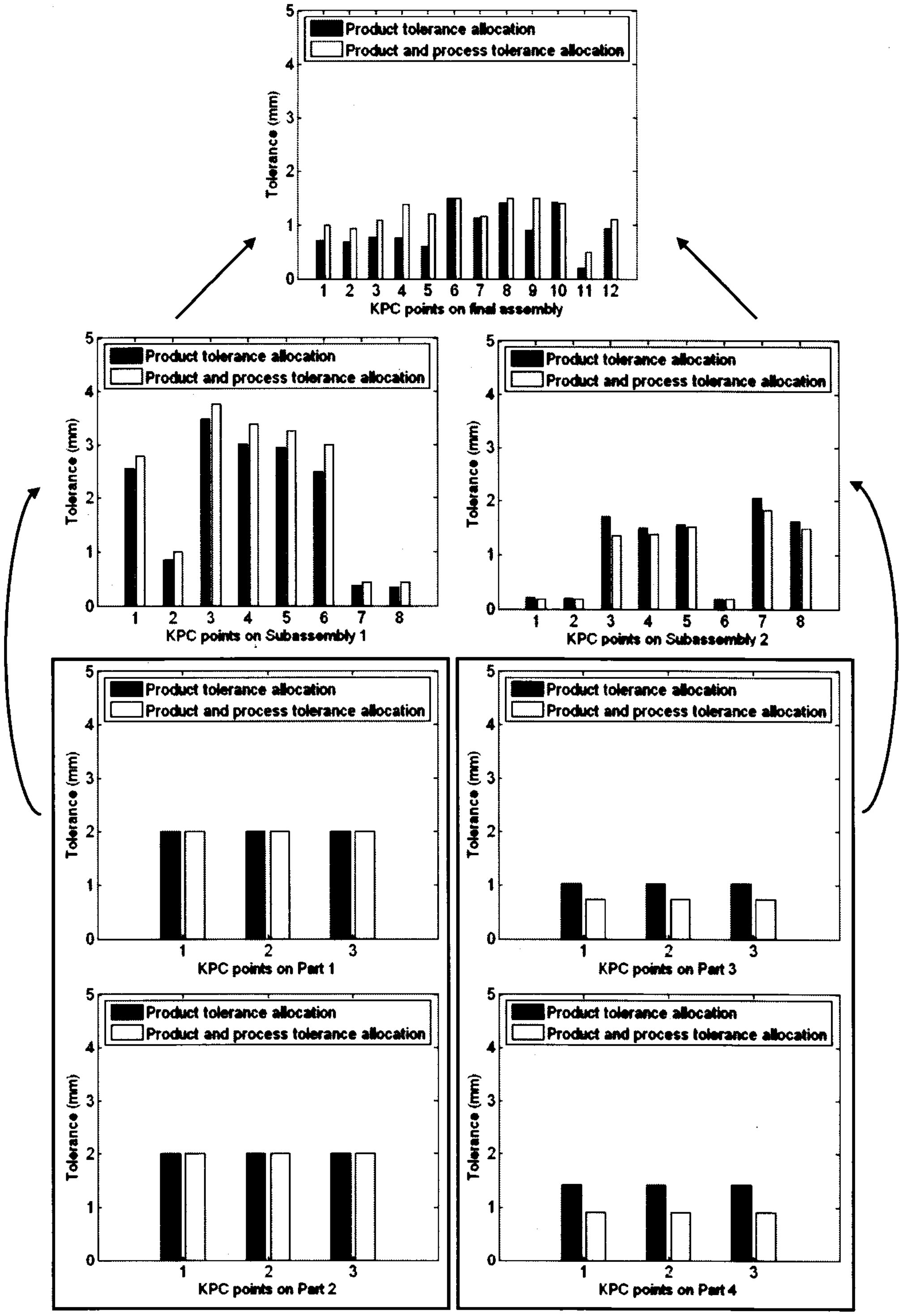

Figure 2.6: Comparison of allocated tolerance values for KPCs of parts, subassemblies, and final assembly, with and without process tolerance allocation

Table 2.1: Results of allocated product tolerances for different assumed tolerances of fixture locators

Case No.	1	2	3	4	5	6	7
	0.0	0.2	0.3	0.4	0.5	0.67	0.67
	0.0	0.2	0.3	0.4	0.5	0.67	0.67
τ_{11}(mm)	0.0	0.2	0.3	0.4	0.5	0.67	0.67
	0.0	0.2	0.3	0.4	0.5	0.67	0.67
	0.0	0.2	0.3	0.4	0.5	0.67	0.67
	0.0	0.2	0.3	0.4	0.5	0.67	0.67
	0.0	0.2	0.3	0.4	0.5	0.18	0.67
	0.0	0.2	0.3	0.4	0.5	0.67	0.67
τ_{12}(mm)	0.0	0.2	0.3	0.4	0.5	0.67	0.67
	0.0	0.2	0.3	0.4	0.5	0.67	0.67
	0.0	0.2	0.3	0.4	0.5	0.48	0.67
	0.0	0.2	0.3	0.4	0.5	0.67	0.67
	0.0	0.2	0.3	0.4	0.5	0.67	0.67
	0.0	0.2	0.3	0.4	0.5	0.67	0.67
τ_{01}(mm)	0.0	0.2	0.3	0.4	0.5	0.66	0.67
	0.0	0.2	0.3	0.4	0.5	0.49	0.67
	0.0	0.2	0.3	0.4	0.5	0.43	0.67
	0.0	0.2	0.3	0.4	0.5	0.67	0.67
	2.0	2.0	2.0	2.0	2.0	2.0	0.61
$\mathbf{t}_{21}$(mm)	2.0	2.0	2.0	2.0	2.0	2.0	0.61
	2.0	2.0	2.0	2.0	2.0	2.0	0.61
	2.0	2.0	2.0	2.0	2.0	2.0	0.41
$\mathbf{t}_{22}$(mm)	2.0	2.0	2.0	2.0	2.0	2.0	0.41
	2.0	2.0	2.0	2.0	2.0	2.0	0.41
	1.03	0.97	0.89	0.77	0.57	0.74	0.04
$\mathbf{t}_{23}$(mm)	1.03	0.97	0.89	0.77	0.57	0.74	0.04
	1.03	0.97	0.89	0.77	0.57	0.74	0.04
	1.42	1.35	1.26	1.11	0.87	0.92	0.07
$\mathbf{t}_{24}$(mm)	1.42	1.35	1.26	1.11	0.87	0.92	0.07
	1.42	1.35	1.26	1.11	0.87	0.92	0.07

are decreasing with increasing variations of fixture locators. When all the process (fixture) tolerances exceed 0.67mm, there are no solutions for product tolerances to satisfy the final product quality requirement.

2.6 Tradeoffs Between System Cost and Final Product Quality

To study the relations between system cost and final product quality, parametric studies are conducted for both cost-driven tolerance allocation and quality-driven tolerance allocation, using different values for quality requirement q_s and budget b, respectively.

The cost-driven product and process tolerance allocation problem is formulated by Problem (2.22). The AIO problem formulation for quality-driven with process tolerance allocation is presented below:

$$\begin{aligned} \min_{\mathbf{t}_{21},\mathbf{t}_{22},\mathbf{t}_{23},\mathbf{t}_{24},\tau_{11},\tau_{12},\tau_{01}} \quad & \|\mathbf{t}_{01}(\mathbf{t}_{21},\mathbf{t}_{22},\mathbf{t}_{23},\mathbf{t}_{24},\tau_{11},\tau_{12},\tau_{01})\|_\infty \\ \text{s.t.} \quad & c_{01}(\mathbf{t}_{21},\mathbf{t}_{22},\mathbf{t}_{23},\mathbf{t}_{24},\tau_{11},\tau_{12},\tau_{01}) \leq b \\ & 0.01\text{mm} \leq \mathbf{t}_{21},\mathbf{t}_{22},\mathbf{t}_{23},\mathbf{t}_{24} \leq 2\text{mm} \\ & 0.01\text{mm} \leq \tau_{11},\tau_{12},\tau_{01} \leq \tfrac{2}{3}\text{mm}. \end{aligned} \tag{2.23}$$

where $\mathbf{t}_{21}$, $\mathbf{t}_{22}$, $\mathbf{t}_{23}$, and $\mathbf{t}_{24}$ are the tolerance vectors of incoming parts, $\mathbf{t}_{01}$ is the tolerance vector of final assembly product, c_{21}, c_{22}, c_{23}, and c_{24} are the costs of incoming parts, c_{01} is the total system cost of the assembly process, and τ_{01}, τ_{11}, and τ_{12} are process tolerance vectors of fixture locators. Please refer to Equations (2.9) and (2.11) for tolerance transfer models and cost-tolerance relations.

The results for cost-driven tolerance allocation problem are presented in Table 2.2, and those for quality-driven tolerance allocation problem are presented in Table 2.3.

Table 2.2: Results for the cost-driven tolerance allocation problem (2.22)

q_s(mm)	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
c	54.5	52.8	51.2	49.7	48.4	47.1	45.9	44.8	43.7	42.7	41.8

Table 2.3: Results for the quality-driven tolerance allocation problem (2.23)

q(mm)	0.97	1.03	1.09	1.15	1.22	1.28	1.35	1.43
b	55	54	53	52	51	50	49	48
q(mm)	1.51	1.59	1.68	1.77	1.87	1.97	2.08	2.20
b	47	46	45	44	43	42	41	40

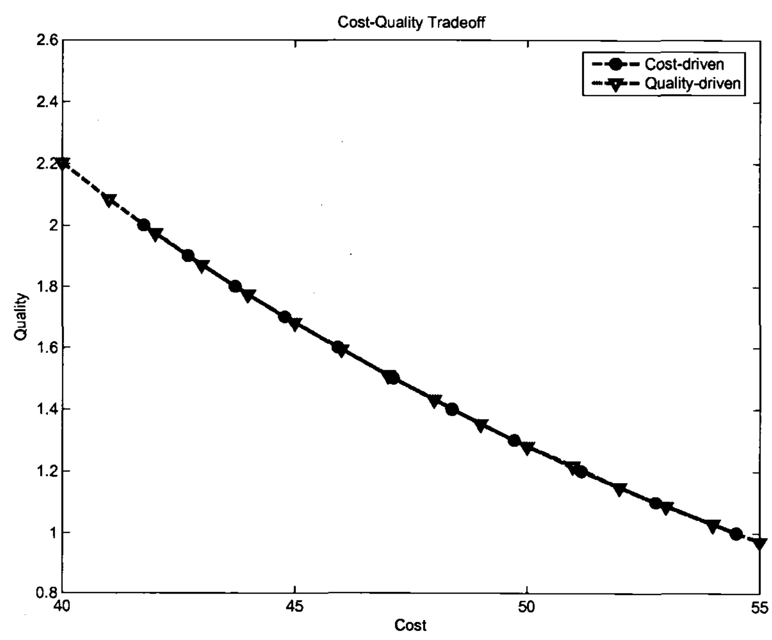

Figure 2.7: Tradeoff between system cost and final product quality

Figure 2.7 is drawn based on data from Table 2.2 and Table 2.3. It is observed that there is a tradeoff between system cost and final product quality. It can also be seen that the constraints $q \leq q_s$ and $c \leq b$ are both active and the two Pareto sets coincide. This implies that the Pareto set can be generated by solving either the cost-driven tolerance allocation problem (2.22) or the quality-driven tolerance allocation problem (2.23).

As mentioned earlier, in the manufacturing literature, the cost-driven problem formulation can be seen as a typical problem formulation for minimum cost tolerance allocation. The quality-driven problem formulation can be seen as a problem formulation for variation reduction. From the above analysis, when q_s and b are chosen appropriately, these two problems should yield the same results for allocated tolerances.

2.7 Comparison of AIO and ATC Strategies

The first four scenarios presented were solved by using both the all-in-one (AIO) strategy and the analytical target cascading (ATC) strategy. The third scenario, the cost-driven product tolerance allocation, is presented below as an example to demonstrate the feasibility of the proposed ATC strategy. Results are compared to the ones obtained by using AIO optimization strategy. Table 2.4 summarizes the tolerance allocation results for the final product measurement points, demonstrating the validity of the ATC strategy.

Table 2.4: Optimal tolerances (mm) for KPCs of final product using AIO and ATC

KPC No.	1	2	3	4	5	6	7	8	9	10	11	12
AIO	0.72	0.69	0.78	0.76	0.60	1.50	1.14	1.41	0.91	1.43	0.20	0.95
ATC	0.72	0.69	0.79	0.76	0.60	1.52	1.15	1.43	0.92	1.45	0.20	1.11

It is emphasized that the ATC strategy is particularly advantageous when considering large-scale problems and complex multistation assembly systems. The problem size of the example used in this work is not large enough to highlight advantages of the ATC methodology over AIO optimization. Nevertheless, the ATC approach demonstrates its advantage over AIO by being able to solve a subproblem of designing the station without redesigning the whole system, while meeting the cascaded targets. Additionally, ATC is able to solve a system of subproblems, which AIO may not be able to solve without decomposition of a large design problem.

2.8 Summary

A multilevel optimization methodology was proposed for product and process tolerance allocation in compliant multistation assembly. Specifically, using variation propagation models, the ATC process was applied to allocate tolerances of KPCs for parts, subassemblies, and final assemblies, and to allocate tolerances of KCCs for locating fixtures. This allows us to identify cost-quality tradeoffs quantitatively. Optimal design specifications for initial parts and subassemblies were obtained to satisfy overall targets related to minimum total costs, or minimum variation for assembled products. The feasibility of the ATC strategy was demonstrated on a compliant multistation assembly example.

The Pareto set representing the tradeoff between cost and quality can be generated by solving either the cost-driven tolerance allocation problem or the quality-driven tolerance allocation problem. The quality-driven tolerance allocation showed that a global (cumulative) budget constraint is preferable to local budget constraints, because it provides the flexibility to allocate resources. The cost-driven tolerance allocation demonstrated that resources should be allocated to more sensitive parts.

Additionally, the consideration of process variations results in different tolerance allocation schemes for products.

CHAPTER III

Optimal Fixture Layout Design

The most important goals of assembly system design are: to increase system robustness, to reduce manufacturing costs, and to satisfy the product quality requirements. The main contributors to product quality and system robustness are variations from incoming parts and variations from tools at each station. The variations from incoming parts could be changed through product tolerance allocation, while the variations from tools could be controlled by assembly sequence, fixture layout design, welding configuration, and process tolerance allocation. Product and process tolerance allocation for multistation assembly systems have been addressed in the previous chapter. Next, related work in optimal fixture layout design will be presented.

Product dimensional variations resulting from locating pins and NC blocks are generally different: variation from locating pins causes a (global) rigid-body motion of a workpiece while variation from NC blocks can cause (local) deformations [Kim 04]. In this dissertation, based on the work of Kim and Ding [Kim 04], the global variation phenomena related to locating pins will be studied for rigid multistation assembly systems. Manufacturing models for compliant assembly systems mainly consider out-of-plane errors, which are part deformations resulting from the NC blocks. In this dissertation, the local variation phenomena related to NC blocks will be explored for compliant multistation assembly systems.

3.1 Problem Description

The objective of fixture layout design is to improve system robustness by changing the fixture positions, given satisfaction of geometric constraints and kinematic constraints. The system robustness describes the sensitivity of dimensional assembly variations with respect to variations of parts, fixtures, and tools.

This goal can be achieved by minimizing the *sensitivity index* (SI), a quantitative measure of fixture layout design, to increase the system robustness. The problem for the fixture layout design in a multistation assembly system can be generally formulated as

$$\begin{aligned} \min_{\mathbf{p}} \quad & SI(\mathbf{p}) \\ \text{s.t.} \quad & \mathbf{g}(\mathbf{p}) \leq \mathbf{0}, \end{aligned} \tag{3.1}$$

where the design variables $\mathbf{p}$ represent fixture locations. The constraints $\mathbf{g}(\mathbf{p}) \leq \mathbf{0}$ are geometrical constraints on fixture locations, imposed by geometries of parts.

As mentioned in [Kim 04], there are three key elements needed in order to solve a multistation fixture layout design problem: a multistation variation propagation model, a sensitivity index definition, and an effective and efficient optimization algorithm. In this dissertation, the sensitivity index will be defined for compliant multistation assembly systems based on compliant multistation variation propagation models [Camelio 03] and the definition of sensitivity index for rigid systems [Kim 04]. The selection of design variables for fixture positions will be explored for compliant systems, in order to limit the computational cost on finite element models with the compliant assembly variation analysis (CAVA). Accordingly, appropriate optimization algorithms will be chosen to be integrated with finite element analysis.

3.2 Sensitivity Index Definition

In this section, a four-station rigid assembly process will be presented, and the variation propagation models for rigid systems will be reviewed. These provide the basis for sensitivity index definition to evaluate different fixture layout designs. Based on these research works, the sensitivity index will be defined for compliant multistation assembly systems.

3.2.1 Rigid Systems

The example and its manufacturing models for rigid multistation assembly systems were provided by Du *et al.* from their class project "Multistation Fixture Layout Optimization for Product Variety" [Du 04]. A four-station assembly process for a family of mid-size passenger sedans is used in the project to illustrate relevant concepts and methodologies.

The two-dimensional rigid body panel assembly model is shown in Figure 3.1. The body side frame panel analyzed consists of four parts: a front wheel housing part (Part 1), a front passenger compartment (Part 2), a rear passenger compartment (Part 3), and a rear quarter panel (Part 4). For simplicity, the parts are modeled as a combination of four quadrilaterals, with dimension presented by the location of the vertices A, B, C, and D. (Please refer to Table 3.1.) The locations of measurement points (MP), where key product characteristics (KPCs) are evaluated, are marked with triangles in the figure. (Please refer to Table 3.2.)

As shown in Figure 3.2, part 1 and part 2 are assembled at station I to form subassembly 1. Subassembly 1 and part 3 are assembled at station II to form

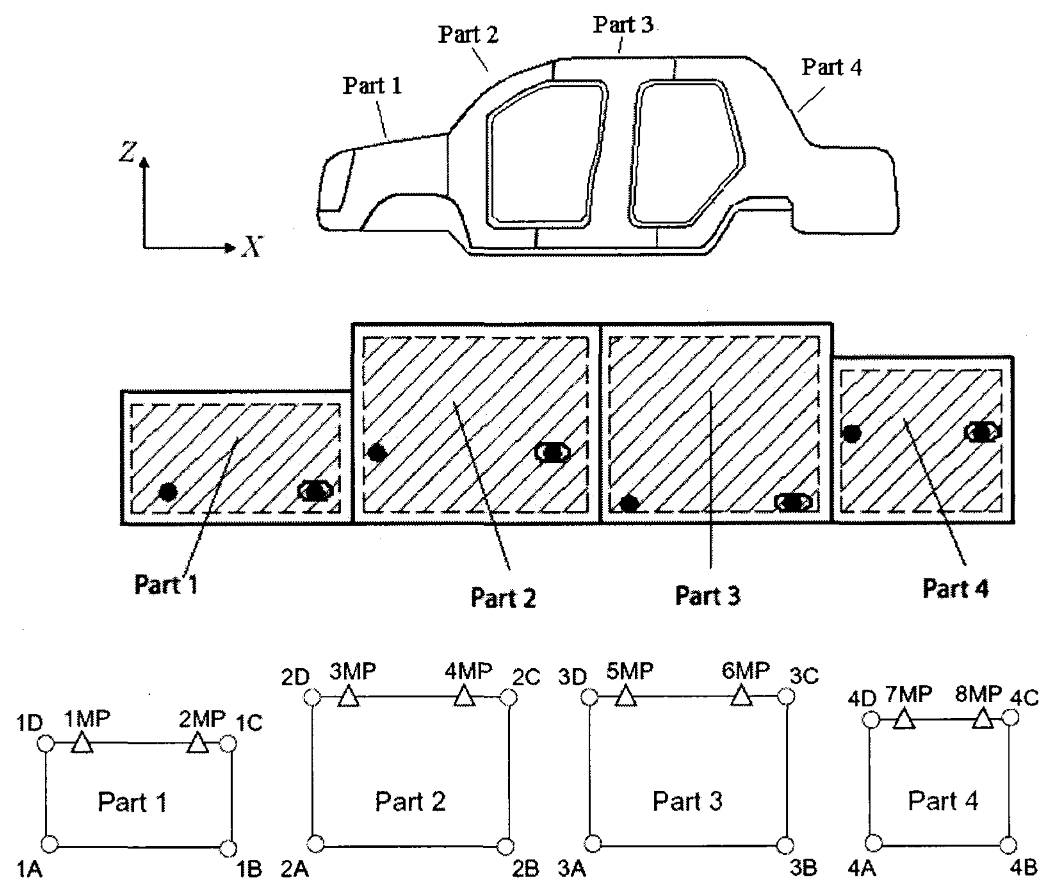

Figure 3.1: Example: right-hand-side auto body subassembly model

Table 3.1: Part dimensions of the rigid multistation assembly example

	Point	X (mm)	Z (mm)
Part 1	1 A	0	0
	1 B	700	0
	1 C	700	400
	1 D	0	400
Part 2	2 A	700	0
	2 B	1450	0
	2 C	1450	600
	2 D	700	600
Part 3	3 A	1450	0
	3 B	2150	0
	3 C	2150	600
	3 D	1450	600
Part 4	4 A	2150	0
	4 B	2700	0
	4 C	2700	500
	4 D	2150	500

Table 3.2: Positions of measurement points on the rigid multistation assembly example

Point	X (mm)	Z (mm)
1 MP	200	400
2 MP	700	400
3 MP	700	600
4 MP	1450	600
5 MP	1550	600
6 MP	2100	600
7 MP	2200	200
8 MP	2700	200

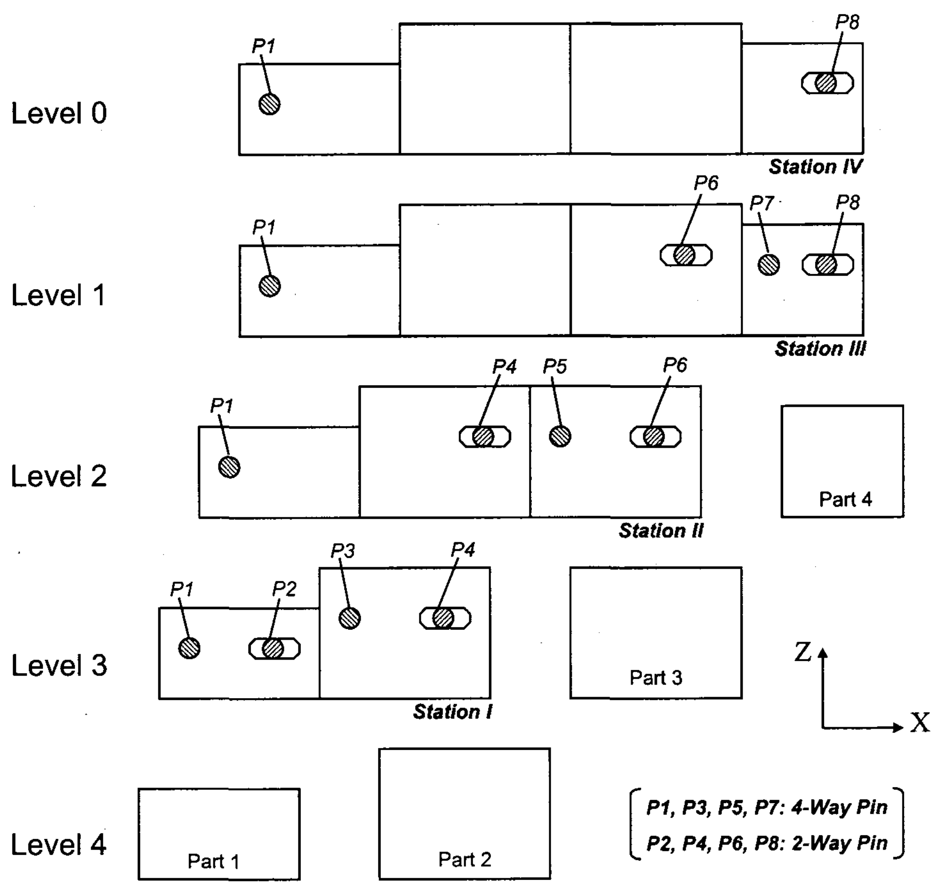

Figure 3.2: Fixture layout in the rigid multistation assembly system

subassembly 2. At station III, subassembly 2 and part 4 are joined together as the final assembly. Then measurement points on the final assembly are inspected at Station IV.

In this study for rigid multistation assembly processes, we focus on the possible variation of locating pins only; thus, $\{P_{4way}, P_{2way}\}$ is used as a simplified representation of a "3-2-1" fixture layout. Locating pins for parts, subassemblies, and final products for the case analyzed are shown on Figure 3.2. The changes in fixture layout along the assembly flow can be represented as follows,

$$\begin{aligned}\{\{P_1, P_2\}, \{P_3, P_4\}\}_I &\rightarrow \{\{P_1, P_4\}, \{P_5, P_6\}\}_{II} \\ &\rightarrow \{\{P_1, P_6\}, \{P_7, P_8\}\}_{III} \\ &\rightarrow \{\{P_1, P_8\}\}_{IV}.\end{aligned}$$

After presenting the geometrical relations, fixture layouts, assembly sequences, and key product and control characteristics, the next step is to link the information to the dimensional variation propagation models. The variation propagation models are reviewed next.

Variation propagation models have been developed for different processes, including rigid-part assembly [Mantripragada 99, Jin 99, Lawless 99], compliant-part assembly [Camelio 03], machining [Djurdjanovic 01, Agrawal 99], and stretch forming [Suri 99].

For rigid multistation assembly systems, Lawless *et al.* [Lawless 99] investigated variation transmission in both assembly and machining processes using time series analysis. This method estimates parameters in the autoregressive models based on tracking the characteristics of individual parts as they pass through multiple stations. Mantripragada and Whitney [Mantripragada 99] proposed a variation propagation model using state transition models, and they applied it to the analysis of a multistation assembly system. The model describes the state space vector by a translation and re-orientation. The mathematical model of the multistation rigid body 2-D assembly was developed by Jin and Shi [Jin 99]. This model is based on the state-space representation of the assembly process; it relies on a standard kinematics analysis of the possible deviations of the parts and their stack up during the process.

The station-indexed state-space variation propagation model [Jin 99, Ding 00a, Ding 02] can be expressed as

$$\mathbf{x}_k = \mathbf{A}_{k-1}\mathbf{x}_{k-1} + \mathbf{B}_k\mathbf{u}_k + \mathbf{w}_k, \text{ and} \tag{3.2}$$

$$\mathbf{y}_k = \mathbf{C}_k\mathbf{x}_k + \mathbf{v}_k, k \in \{1, 2, \ldots, N\}, \tag{3.3}$$

where $\mathbf{x}_k$ represents the deviations of parts at station k, $\mathbf{u}_k$ is the tooling deviation, and $\mathbf{y}_k$ represents the deviations of assembly KPCs. $\mathbf{w}_k$ is the unmodeled process deviation, such as the higher order terms resulting from linearization or other variation sources. $\mathbf{v}_k$ represents the additive sensor noise. $\mathbf{w}_k$ and $\mathbf{v}_k$ are assumed mutually independent. N is the number of stations.

The first equation is known as the state equation, which implies that the part deviation at station k is influenced by the deviation propagated to station $(k-1)$, and the deviation contribution from station k. The second equation is the observation equation.

State matrices $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ depend on fixture layout change in a multistation assembly system. Matrix $\mathbf{A}_k$ characterizes the assembly reorientation during part transfer between stations, and it links product dimensional states $\mathbf{x}$ across different stations. Matrix $\mathbf{A}_k$ depends on station-to-station fixture layout change in a production stream. Matrix $\mathbf{B}_k$ models the effect of fixture variation $\mathbf{u}_k$ on the product dimensional state $\mathbf{x}_k$. Matrix $\mathbf{C}_k$ includes the information about sensor positions on the product, which are often the measurement points selected during design stage. Usually, KPCs are evaluated at these measurement points on the final product, since they correspond to the end-of-line observation in the previously presented four-station assembly process (Figure 3.2). In this rigid assembly process, $\mathbf{C}_{1,2,3} = \mathbf{0}$ and $\mathbf{C}_4 \neq \mathbf{0}$ because KPCs are measured on Station IV only after assembly operations on Stations I, II, and III.

The importance of the multistation assembly model, is its capacity to predict the variation of the KPCs, taking into account the disturbances, the noises, the variation of the fixtures locations, the incoming part variations, and the variation propagation due to the relocation of the subassemblies at downstream stations.

For the rigid multistation assembly process, Kim and Ding [Kim 04] reformulated the state-space model into a linear model by eliminating all intermediate state variables $\mathbf{x}_k$,

$$\mathbf{y}_N = \sum_{k=1}^{N} \mathbf{C}_N \mathbf{\Phi}_{N,k} \mathbf{B}_k \mathbf{u}_k + \mathbf{C}_N \mathbf{\Phi}_{N,0} \mathbf{x}_0 + \sum_{k=1}^{N} \mathbf{C}_N \mathbf{\Phi}_{N,k} \mathbf{w}_k + \mathbf{v}_N, \tag{3.4}$$

where $\mathbf{\Phi}_{k,i} \equiv \mathbf{A}_{k-1}\mathbf{A}_{k-2} \ldots \mathbf{A}_i$ and $\mathbf{\Phi}_{i,i} \equiv \mathbf{I}$.

Assuming there are no measurement and process noises, and no error from incoming parts, i.e., $\mathbf{w}_k = \mathbf{0}, \mathbf{v}_N = \mathbf{0}$, and $\mathbf{x}_0 = \mathbf{0}$. The above equation can be simplified by focusing on the first term, $\sum_{k=1}^{N} \mathbf{C}_N \mathbf{\Phi}_{N,k} \mathbf{B}_k \mathbf{u}_k$, which represents fixture error inputs from all N stations. The simplified model can be stated as

$$\hat{\mathbf{y}} \equiv \mathbf{D}\mathbf{u} = \sum_{k=1}^{N} \mathbf{C}_N \mathbf{\Phi}_{N,k} \mathbf{B}_k \mathbf{u}_k, \tag{3.5}$$

where $\mathbf{D} \equiv [\mathbf{C}_N \mathbf{\Phi}_{N,1} \mathbf{B}_1,\ \mathbf{C}_N \mathbf{\Phi}_{N,2} \mathbf{B}_2,\ \ldots,\ \mathbf{C}_N \mathbf{B}_N]$,$\mathbf{u}^T \equiv [\mathbf{u}_1^T,\ \ldots,\ \mathbf{u}_N^T]$, and $\hat{\mathbf{y}}$ is the fixture-induced product variation. Note that subscript N is dropped from $\hat{\mathbf{y}}$

hereafter without causing ambiguity. For the rigid multistation assembly example (Figure 3.2), with the assumption $\mathbf{u}_4 = \mathbf{0}$, $\mathbf{D} \equiv [\mathbf{C}_4\mathbf{\Phi}_{4,1}\mathbf{B}_1,\ \mathbf{C}_4\mathbf{\Phi}_{4,2}\mathbf{B}_2,\ \mathbf{C}_4\mathbf{\Phi}_{4,3}\mathbf{B}_3]$.

One way to define the sensitivity index is as follows,

$$S \equiv \frac{\hat{\mathbf{y}}^T\hat{\mathbf{y}}}{\mathbf{u}^T\mathbf{u}} = \frac{\mathbf{u}^T\mathbf{D}^T\mathbf{D}\mathbf{u}}{\mathbf{u}^T\mathbf{u}}, \tag{3.6}$$

where $\hat{\mathbf{y}}^T\hat{\mathbf{y}} = \mathbf{u}^T\mathbf{D}^T\mathbf{D}\mathbf{u}$ is the sum of squares of product deviations. This benchmarks the overall level of product dimensional nonconformity, as well as the product quality.

The goal was to define a sensitivity index that is determined only by fixture design information (modeled by $\mathbf{D}$), and that is independent of variation input (represented by $\mathbf{u}$). Among several measures related to $\mathbf{D}^T\mathbf{D}$, E-optimality is chosen as an informative criterion for multistation fixture layout design. E-optimality serves to minimize the extreme eigenvalue $\lambda_{max}(\mathbf{D}^T\mathbf{D})$ and is equivalent to minimizing the upper sensitivity boundary of the fixture system. This criterion can also be defined using the concept of matrix 2-norm, i.e, the E-optimal condition is the square of the 2-norm of design matrix $\mathbf{D}$. In summary, the sensitivity index used in multistation fixture layout design is defined as

$$SI \equiv S_{max} \equiv \sup_{\mathbf{u}\neq 0}\frac{\mathbf{u}^T\mathbf{D}^T\mathbf{D}\mathbf{u}}{\mathbf{u}^T\mathbf{u}} = \|\mathbf{D}\|_2^2 = \lambda_{max}(\mathbf{D}^T\mathbf{D}). \tag{3.7}$$

3.2.2 Compliant Systems

Multistation variation propagation models for compliant systems were extensively reviewed in Section 2.2.1. For the multistation assembly system under consideration, as shown in Figure 3.3, the fixture scheme is "3-2-1" rather than "N-2-1" ($N > 3$) and welding guns are perfect. The variation propagation model is then simplified as,

$$\mathbf{x}_k = \mathbf{S}_{\mathbf{A}_k}\mathbf{x}_{k+1} + \mathbf{S}_{\mathbf{B}_k}\mathbf{u}_{k1}, \tag{3.8}$$

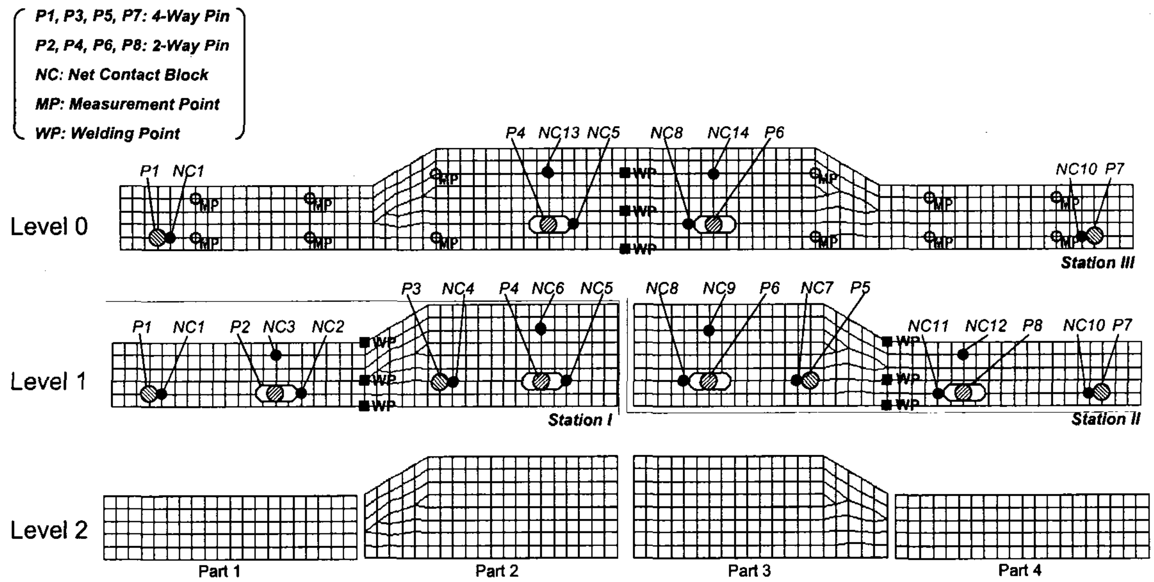

Figure 3.3: Fixture layout in the compliant multistation assembly system

where $\mathbf{x}_k$ is the dimensional variation state vector that corresponds to dimensional variations of measurement points on the subassemblies or final assembly. The input vectors includes the dimensional variation state vectors of the component KPCs $\mathbf{x}_{k+1}$ and the "3-2-1" locating fixtures $\mathbf{u}_{k1}$. $\mathbf{S}_{\mathbf{A}_k}$ and $\mathbf{S}_{\mathbf{B}_k}$ can be obtained by applying compliant assembly variation analysis (CAVA) [Hu 00] on finite element models.

Assuming that tolerance t is represented by standard deviation σ_x according to $t = 6\sigma_x$, the tolerance transfer model is derived as,

$$\mathbf{t}_k = \mathbf{F}_{t_k}(\mathbf{t}_{k+1}, \tau_k) = \sqrt{\mathbf{S}^2_{\mathbf{A}_k}\mathbf{t}^2_{k+1} + \mathbf{S}^2_{\mathbf{B}_k}\tau^2_k}, \quad (3.9)$$

where the process tolerances τ are the tolerances associated with KCCs. The product tolerances $\mathbf{t}$ are the tolerances associated with KPCs. Next, the sensitivity index will be defined based on the tolerance transfer model.

For a single input-output pair, the sensitivity can be defined as $S_{ij} = \frac{y_i}{x_j}$, where y_i is the ith product feature and x_j is the jth error input. For the entire assembly system with multiple inputs and multiple features, the sensitivity can be defined as,

$$S = \frac{\sqrt{\mathbf{y}^T\mathbf{y}}}{\sqrt{\mathbf{x}^T\mathbf{x}}} = \frac{||\mathbf{y}||}{||\mathbf{x}||}. \tag{3.10}$$

Accordingly, the sensitivity index SI in a compliant system can be defined as the norm of a tolerance output vector $\mathbf{t}_0$ with respect to the norm of unit product and process tolerance input vectors,

$$\begin{aligned} SI &= ||\mathbf{t}_0||, \\ \mathbf{t}_k &= \mathbf{F}_{t_k}(\mathbf{t}_{k+1}, \tau_k), \;\; k \in 0, 1, 2, \ldots, N-1, \\ \hat{\mathbf{t}_N} &= \mathbf{1}, \text{ and} \\ \hat{\tau_k} &= \frac{\mathbf{1}}{\mathbf{3}}, \;\; k \in 1, 2, \ldots, N-1. \end{aligned} \tag{3.11}$$

The process tolerance value is set as one-third of the product tolerance value. The sensitivity matrices in tolerance transfer model $\mathbf{F}_{t_k}$ depend on fixture positions $\mathbf{p}$ and are obtained using CAVA on finite element models. Compared to the sensitivity definition for rigid systems, the definition for compliant systems takes into account the effects of product variations from incoming parts and subassemblies.

3.3 Selection of Design Variables

Section 1.2.2 mentioned that, for the fixture layout design in compliant assembly systems, the use of finite element models to obtain sensitivity matrices in variation propagation models increases the computational cost and the complexity of the fixture layout optimization. Also, the fixture layout design in compliant assembly systems is more concerned with the integration of optimization algorithms with finite element analysis (FEA) models.

In current literature, considering the effects of fixture variation, both remeshing algorithms and multipoint constraint (MPC) are necessary to calculate the assembly variation for different fixture positions [Camelio 04]. A remeshing algorithm can be used to redefine the nodes and locate the fixtures in the manufacturing model and the optimization algorithm [Rearick 93]. MPC, one of the advanced features in Nastran [MSC.NASTRAN 01], is used to calculate sensitivity matrices in variation propagation models [Cai 96a]. But FEA with the application of both methods results in high computational cost for multistation assembly fixture layout design. It is then necessary to select appropriate design variables in order to limit the computational cost on finite element models with CAVA [Hu 00].

In this dissertation, based on the understanding of manufacturing models, Grid (Node) identification (ID) numbers in finite element models are selected as discrete design variables for fixture location information in compliant systems. This is coordinated with the available CAVA software, because input files of CAVA also need the information of Grid IDs that position measurement points, fixturing points, welding points, and releasing points. The selection can reduce the number of design variables, because each ID represents three dimensions and locates a fixture position. Additionally, using Grid IDs allows the avoidance of the use of MPC and remeshing FEA models; this saves the computation time that would have been spent on manufacturing models, and makes it possible to integrate FEA with optimization processes. Although the fixtures can be located discretely only on the surface, that is acceptable provided there is sufficient FEA mesh density. Accordingly, it is necessary to use mixed-discrete optimization algorithms that allow discrete design variables.

3.4 Design Models

The mathematical formulation for optimal fixture layout design in a *rigid multi-station assembly system* is as follows:

$$\begin{aligned} \min_{\mathbf{p}} \quad & SI(\mathbf{p}) \\ \text{s. t.} \quad & \mathbf{g}(\mathbf{p}) \leq \mathbf{0}, \end{aligned} \tag{3.12}$$

where $\mathbf{p}$ represents the fixture locations, or the locations of principal locating points (PLP), where $\mathbf{p} = [X_1, Z_1, \ldots, X_{n_{PLP}}, Z_{n_{PLP}}]^T$. n_{PLP} is the total number of PLPs. X_k and Z_k are coordinates for kth PLP. $\mathbf{g}(\mathbf{p}) \leq \mathbf{0}$ are geometrical constraints on PLP locations, imposed by geometries of parts. As shown in Figure 3.6, regions inside the dashed lines are the feasible design regions. With any assigned values for $\mathbf{p}$ that satisfy the geometrical constraints, it is then possible to evaluate SI [Kim 04]. The optimization process is presented in Figure 3.4.

The mathematical formulation for optimal fixture layout design in a *compliant multistation assembly system* is as follows:

$$\begin{aligned} \min_{\mathbf{p}} \quad & SI(\mathbf{p}) \\ \text{s. t.} \quad & \mathbf{g}(\mathbf{p}) \leq \mathbf{0}, \end{aligned} \tag{3.13}$$

where $\mathbf{p}$ represents the fixture locations by using the grid identification number in the finite element analysis (FEA) input file. In FEA, each grid identification number corresponds to the three coordinates, X, Y, and Z of a grid node. $\mathbf{g}(\mathbf{p}) \leq \mathbf{0}$ are geometrical constraints on fixture locations, imposed by geometries of parts and the positions of welding points.

In this work for compliant systems, four-way pins and two-way pins are reused for the downstream stations, and are not included in the fixture design. NC blocks

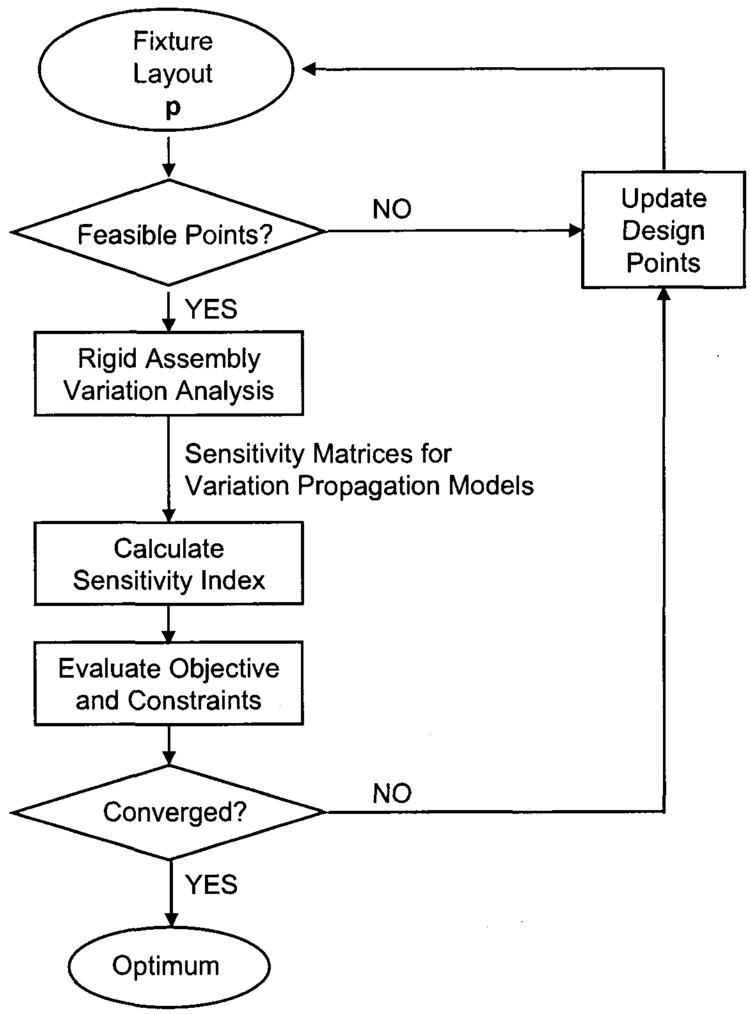

Figure 3.4: Optimization process for fixture layout design in rigid multistation assembly systems

coupled with four-way pins and two-way pins are also fixed ($NC_{1,2,4,5,7,8,10,11}$ in Figure 3.3). The fixtures to be designed are $NC_{3,6}$ at station I, $NC_{9,12}$ at station II, and $NC_{13,14}$ at station III.

Since the geometrical constraint is the only constraint in the design, the feasible region can be defined before optimization processes start. The procedure presented below is based on the understanding of finite element models and design models. To find the set of feasible Grid IDs for one station with two workpieces:

1. Read the Hypermesh model file; save all Grid IDs.
2. Separate IDs into two groups as they relate to two workpieces.
3. Remove welding point IDs.
4. Remove boundary point IDs because fixtures cannot be located on the boundaries of workpieces.
5. Remove other infeasible point IDs, such as those too close to the fixtures, the boundaries, and the welding points.

The optimization process is then presented in Figure 3.5.

3.5 Optimization Algorithms

Gradient-based optimization algorithms such as sequential quadratic programming (SQP) [MATLAB 04], are widely used in solving fixture design problems. The gradient-based methods tend to converge quickly, but they can be easily entrapped in local optimizers. Thus global optimality becomes difficult, especially for a high-dimensional design space in multistation assembly systems.

Derivative-free optimization algorithms can be applied, but they may be computationally expensive. DIvided RECTangles (DIRECT), and Multi-Island Genetic Algorithm (MIGA), have been chosen for this work.

The DIRECT optimization algorithm [Jones 01] can solve mixed-integer nonlinear programming problems. The global optimum can be located efficiently without derivative information, when the number of variables is small. DIRECT starts at the center of the user-supplied design space, divides it into rectangles, and evaluates the objective function at the center points of these rectangles. Based on the objective

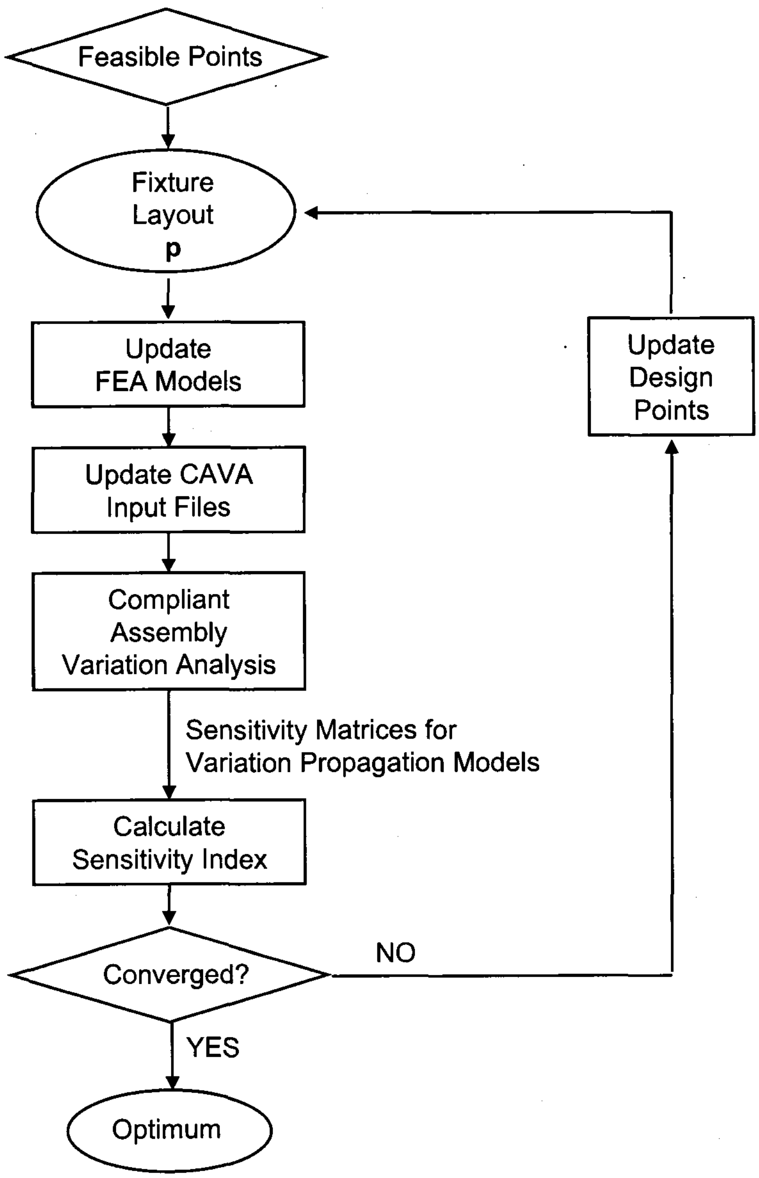

Figure 3.5: Optimization process for fixture layout design in compliant multistation assembly systems

function value and the characteristic dimension associated with each rectangle, DIRECT selects which rectangles to divide further until it reaches the specified number of function evaluations. This ensures that the entire space is examined with sufficient granularity in order to explore more promising areas in more detail.

In the Multi-Island Genetic Algorithm (MIGA), as with other genetic algorithms, each design point is perceived as an individual, with a certain value of fitness based on the value of objective function and constraint penalty. Here the algorithm used was the one implemented in the iSIGHT software package [iSIGHT 04]. An individual with a better value of objective function and penalty has a higher fitness value. Each individual is represented by a chromosome in which the values of design variables are converted into a binary string of zeroes and ones. This conversion is called "encoding" the individual. Each population of individuals (a set of design points) is altered via the genetic operations of "selection," "crossover," and "mutation." Each design of a population is evaluated by iSIGHT, and its fitness value is determined. A new population of designs is selected from the original set of designs. The main feature of MIGA that distinguishes it from traditional genetic algorithms is the fact that each population of individuals is divided into several sub-populations, which are called "islands." All traditional genetic operations are performed separately on each sub-population. Some individuals are then selected from each island and periodically migrated to different islands. This operation is called "migration."

3.6 Results

3.6.1 Fixture Layout Design for Rigid Systems

In this work, derivative-free optimization algorithms are used to find an optimal solution globally. The solution serves as the initial condition for gradient-based optimization algorithms. This fixture layout optimization problem is first solved by DIRECT with 10,000 function evaluations, and then solved by SQP, which uses the previous solution as a starting point. The problem is also solved by MIGA with 10,000 function evaluations, and then solved by SQP, using the previous solution as a starting point. The results are presented in Table 3.3, and are depicted by Figure 3.6.

Table 3.3: Optimal fixture layout design for the rigid multistation assembly system

p	DIRECT only	DIRECT with SQP	MIGA only	MIGA with SQP
P1 x	427.92	421.31	386.67	386.66
P1 z	291.79	286.74	280.85	280.8
P2 x	491.10	497.58	482.77	482.73
P2 z	92.7	88.77	75.11	75.21
P3 x	857.71	858.74	961.07	961.07
P3 z	624.99	625	624.80	625
P4 x	1559.6	1560	1558.6	1558.7
P4 z	220.44	221.45	362.56	362.54
P5 x	1823.7	1824.5	1886.7	1886.7
P5 z	625	625	624.6	625
P6 x	2326.7	2329.7	2325.7	2325.7
P6 z	220.4	224.3	382.98	382.97
P7 x	2835.1	2836.3	2885.9	2885.9
P7 z	81.71	78.63	36.478	36.45
P8 x	2626.6	2626.2	2655.2	2655.2
P8 z	489.35	515	514.13	514.45
Sensitivity Index	10.887	10.859	10.841	10.840

It can be observed that the integration of derivative-free and gradient-based optimization algorithms is effective in reaching a better solution than that individual algorithms can provide. Among the four strategies described above, MIGA with SQP provides a solution with a marginally lower sensitivity index, i.e., better system robustness.

It can also be seen that all strategies yielded essentially similar values for SI.

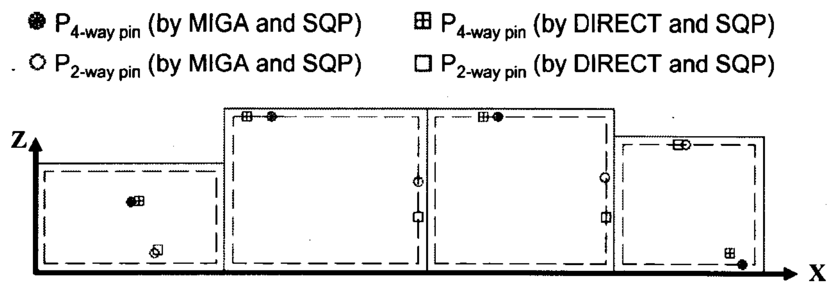

Figure 3.6: Optimal fixture layout for the rigid multistation assembly system

Comparing the best available result 10.840 generated by MIGA with SQP, and the worst result 10.887 generated by DIRECT, there is only 0.4-percent difference. But there are substantially different optimal values for some of the design variables. This implies that there exist various solutions of fixture positions which can provide similar SI values close to the global optimum. Additional information, such as the sizes of pins and holes, the shape of the workpiece, and the available positions for fixtures, should be considered to make the final decisions.

3.6.2 Fixture Layout Design for Compliant Systems

Gradient-based optimization algorithms are not appropriate for solving the fixture layout optimization problems for compliant multistation assembly systems. This problem is thus solved by DIRECT algorithm, and Multi-Island Genetic Algorithm (MIGA). The results are presented in Table 3.4, and are depicted by Figure 3.7.

It can be observed that all strategies yielded similar values for SI. Comparing the best available solution 4.655 generated by DIRECT with 10,000 function evaluations, and the worst result 4.741 generated by MIGA with 2,200 function evaluations, the difference is less than two-percent. Considering the computational cost of simulations for compliant systems is much more expensive than that for rigid systems, early inclusion of additional information in the design process, such as the sizes of pins and

Table 3.4: Optimal fixture layout design for the compliant multistation assembly system

	DIRECT	DIRECT	MIGA
No. of Function Evaluation	2,500	10,000	2,200
NC_{13} (Station 01)	44	41	39
NC_{14} (Station 01)	28	23	24
NC_3 (Station 11)	11	10	9
NC_6 (Station 11)	31	34	40
NC_9 (Station 12)	7	7	6
NC_{12} (Station 12)	7	5	5
Sensitivity Index (2-norm)	4.720	4.655	4.741

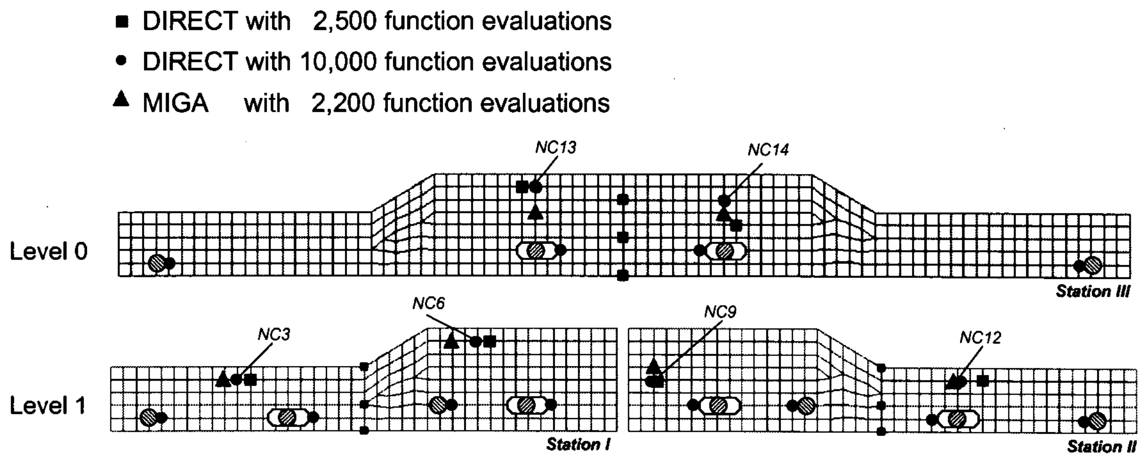

Figure 3.7: Optimal fixture layout for the compliant multistation assembly system

holes, the shape of the workpiece, and the available positions for fixtures, is necessary to make the final decisions with relatively short computational time. Additionally, as observed from Figure 3.7, fixture locations are close to each other. This may enable further improvement of optimization efficiency by using derivative-free optimization algorithms to find a good initial guess for more efficient gradient-based optimization algorithms.

3.7 Summary

The optimal fixture layout design methodology for compliant multistation assembly systems allows increasing the system robustness, with a focus on the impact of fixture position on the dimensional quality of compliant parts after assembly. The methodology considers part and tooling variation and assembly springback. A sensitivity index was defined as a quantitative measure to evaluate the fixture layout design using multistation variation propagation models. Grid (Node) IDs in finite element models were selected as discrete design variables of fixture locations. Mixed-discrete derivative-free optimization algorithms (DIRECT and MIGA) were integrated with finite element analysis to find the optimal fixture positions, such that the sensitivity index is minimized.

Rearick *et al.* [Rearick 93] proposed a simple cost model to evaluate the cost of additional fixtures and the deviations for final assembly. In this work, it is assumed that there is no extra cost for setting up or maintaining a robust assembly system. The robustness is improved only by changing the fixture positions.

The traditional thought about fixture layout optimization is that an optimal fixture layout design improves the robustness of an assembly system, reduces product variability, and leads to manufacturing cost reduction. Is this always true when the cost and the robustness are optimized at the same time, even under the assumption that no extra cost is needed to maintain a robust system? To answer this question, tolerance allocation and fixture layout design are considered simultaneously for multistation assembly systems in the next chapter.

CHAPTER IV

Tolerance Allocation with Fixture Layout Design

The previous chapters considered tolerance allocation and fixture layout design independently. This chapter considers tolerance allocation and fixture layout design simultaneously. Multiobjective problems will be formulated to examine the relations among system cost, final product quality, and system robustness. As the problem size of the integrated design becomes larger, having more design variables, and the simulation cost increases because of the involvement of more manufacturing models, appropriate algorithms become necessary to enable solving these multiobjective problems. The traditional thoughts about the relations between cost and robustness, and the relations between quality and robustness, will be reconsidered by the analysis of observed tradeoffs. Parametric studies will be provided to address the effects of constraint activity and design boundary on the design of multistation assembly systems.

4.1 Problem Description

As presented in Section 1.3, Table 4.1 lists the design problems for multistation assembly systems. Problems A, B, and F were addressed in Chapter II. Problem C was addressed in Chapter III. In this chapter, we will solve Problems D, E, and G,

Table 4.1: Design problems for multistation assembly systems

Design Problem	Description	Cost	Quality	SI
A	Cost-driven Tolerance Allocation	Objective	Parameter	Parameter
B	Quality-driven Tolerance Allocation	Parameter	Objective	Parameter
C	Optimal Fixture Layout Design	N/A	N/A	Objective
D	Cost-robustness Relations with Fixed Quality Requirement	Objective	Parameter	Objective
E	Quality-robustness Relations with Fixed Cost Requirement	Parameter	Objective	Objective
F	Cost-Quality Tradeoff with Fixed Fixture Layout	Objective	Objective	Parameter
G	Relations among Cost, Quality, and robustness	Objective	Objective	Objective

and answer these questions:

- How can design problems be formulated when considering tolerance allocation and fixture layout design simultaneously?
- How can appropriate optimization algorithms be chosen when the problem scale is increasing?
- Does a robust system, with a low sensitivity index, really save money?
- What are possible strategies to improve a multistation assembly system?

4.2 Relations Between Cost and Robustness

It is widely thought that an optimal fixture layout design can both increase the system robustness and reduce manufacturing cost by minimizing a sensitivity index. This work reexamines this established thinking by formulating a two-objective problem. Nested optimization strategy will be applied to solve the problem efficiently and effectively. Finally, the results will be discussed in the context of the relations between cost and robustness.

4.2.1 Problem Formulation

The relations between system cost and system robustness are studied by solving a two-objective optimization problem. The problem is formulated as follows based on the cost-driven tolerance allocation problem (2.16) and the optimal fixture layout design problem (3.1):

$$\begin{aligned} \min_{\mathbf{t},\tau,\mathbf{p}} \quad & \{c(\mathbf{t},\tau), SI(\mathbf{p})\} \\ \text{s.t.} \quad & q(\mathbf{t},\tau,\mathbf{p}) \leq q_s \\ & \mathbf{g}(\mathbf{t},\tau) \leq 0 \\ & \mathbf{g}(\mathbf{p}) \leq \mathbf{0}, \end{aligned} \tag{4.1}$$

Here the design variables $\mathbf{t}$ and τ are product and process tolerance design vectors, respectively. The design variables $\mathbf{p}$ are the fixture locations, c is the total cost in the assembly processes, depending only on $\mathbf{t}$ and τ, SI is the sensitivity index, which can be evaluated by variation propagation models, given fixture locations $\mathbf{p}$, and q is the final product quality.

The constraints $\mathbf{g}(\mathbf{p}) \leq \mathbf{0}$ are geometrical constraints on fixture locations, and are imposed by geometries of parts; $\mathbf{g}(\mathbf{t},\tau) \leq 0$ are inequality constraints representing lower and upper design bounds for $\mathbf{t}$ and τ; and q_s is the quality requirement. The constraint $q(\mathbf{t},\tau,\mathbf{p}) \leq q_s$ ensures the final product quality will satisfy the quality requirement.

4.2.2 Nested Optimization Strategy

The complexity of Problem (4.1) poses difficulties for design optimization, especially with the expensive finite element model simulations of compliant systems. Nested optimization strategy is applied based on the understanding of non-gradient optimization algorithms, constraint activities, and manufacturing model characteristics, to improve the result accuracy and optimization process efficiency.

Problem (4.1)is partially separable and so it can also be written as

$$\begin{aligned} \min_{\mathbf{p}} \quad & \{\min_{\mathbf{t},\tau}\{c(\mathbf{t},\tau)|q(\mathbf{t},\tau,\mathbf{p}) \leq q_s, \mathbf{g}(\mathbf{t},\tau) \leq 0\}, SI(\mathbf{p})\} \\ \text{s.t.} \quad & \mathbf{g}(\mathbf{p}) \leq \mathbf{0}. \end{aligned} \tag{4.2}$$

The purpose of the *outer loop* is to determine values for fixture layout design variables $\mathbf{p}$. Given fixture locations $\mathbf{p}$, the sensitivity matrices are obtained by applying manufacturing models. These sensitivity matrices are used as parameters in variation propagation models for the inner loop optimization process. For the *inner loop*, given the variation propagation models and cost models, product and process tolerances, $\mathbf{t}$ and $\boldsymbol{\tau}$, will be allocated to reach the minimum cost c while satisfying the quality requirement q_s. The process is shown in Figure 4.1.

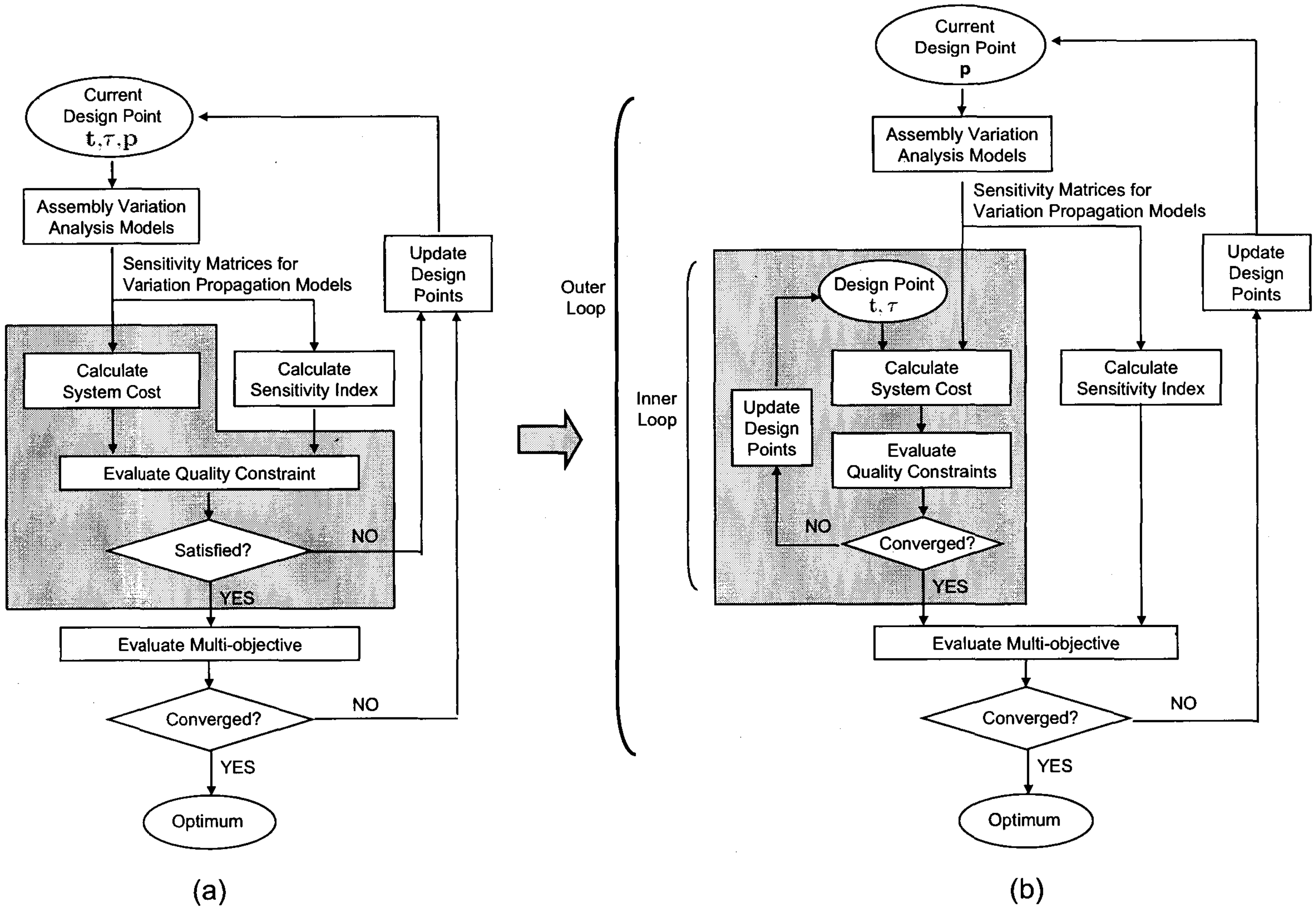

Figure 4.1: Nested optimization strategy for Problem (4.1)

Problem (4.1) and Problem (4.2) are equivalent for evaluating cost-SI relations because the two problem formulations can provide the same objective values for any feasible design point $\mathbf{p}$, per the following discussion.

If $\mathbf{p}$ is a feasible design point in Problem (4.1), we have

$$q(\mathbf{t}, \tau, \mathbf{p}) \leq q_s \text{ and } \mathbf{g}(\mathbf{p}) \leq \mathbf{0}.$$

Therefore, $\mathbf{p}$ is also a feasible design point for Problem (4.2).

Given any feasible design point $\mathbf{p}$ in both problems, it is assumed that $\bar{\mathbf{t}}$ and $\bar{\tau}$ solve the inner tolerance allocation design problem for Problem (4.2). Now, suppose $\bar{\mathbf{t}}$ and $\bar{\tau}$ are not optimal for Problem (4.1). There then exist $\hat{\mathbf{t}}$ and $\hat{\tau}$ such that

$$c(\hat{\mathbf{t}}, \hat{\tau}) \leq c(\bar{\mathbf{t}}, \bar{\tau}),$$

which implies, for a multi-objective value,

$$[c(\hat{\mathbf{t}}, \hat{\tau}), SI(\mathbf{p})] \leq [c(\bar{\mathbf{t}}, \bar{\tau}), SI(\mathbf{p})].$$

Therefore, $\hat{\mathbf{t}}$ and $\hat{\tau}$ are solutions that satisfy the quality constraint $q(\hat{\mathbf{t}}, \hat{\tau}, \mathbf{p}) \leq q_s$, and provide a lower value for system cost c. This contradicts the assumption that $\bar{\mathbf{t}}$ and $\bar{\tau}$ solve the inner tolerance allocation design problem, which should provide the minimum system cost at a certain fixture layout $\mathbf{p}$.

Now, suppose there exist $\hat{\mathbf{t}}$ and $\hat{\tau}$ such that

$$c(\hat{\mathbf{t}}, \hat{\tau}) > c(\bar{\mathbf{t}}, \bar{\tau}),$$

which implies, for multi-objective value,

$$[c(\hat{\mathbf{t}}, \hat{\tau}), SI(\mathbf{p})] > [c(\bar{\mathbf{t}}, \bar{\tau}), SI(\mathbf{p})],$$

which contradicts optimality of $\hat{\mathbf{t}}$ and $\hat{\tau}$.

Gradient-based optimization algorithms can be selected to solve the tolerance allocation problem in the inner loop. Derivative-free optimization algorithms should be used for the outer loop, such as Neighborhood Cultivation Genetic Algorithm (NCGA) from iSIGHT [iSIGHT 04], where optimization processes rely mainly on function evaluations. In NCGA, each objective parameter is treated separately. Standard genetic operations of mutation and crossover are performed on the designs. The crossover process is based on the "neighborhood cultivation" mechanism, where the crossover is performed mostly between individuals with values close to one of the objectives. By the end of the optimization run, a Pareto set is constructed where each design has the "best" combination of objective values, and where improving one objective is impossible without sacrificing one or more of the other objectives.

The following example is solved to show the advantage of nested optimization strategy:

$$\begin{aligned} \min_{\mathbf{t},\tau,\mathbf{p}} \quad & \{c(\mathbf{t},\tau), SI(\mathbf{p})\} \\ \text{s.t.} \quad & q(\mathbf{t},\tau,\mathbf{p}) \le 2\text{mm} \\ & \mathbf{g}(\mathbf{t},\tau) \le 0 \\ & \mathbf{g}(\mathbf{p}) \le \mathbf{0}. \end{aligned} \tag{4.3}$$

Here the design variables $\mathbf{t}$ and τ are product and process tolerance design vectors, respectively. The design variables $\mathbf{p}$ represent the fixture locations; c is the system cost in assembly processes, depending only on $\mathbf{t}$ and τ; SI is the sensitivity index, which can be evaluated by variation propagation models, given fixture locations $\mathbf{p}$; and q is the product quality, representing the maximum value in the tolerance vector of final product.

The constraint $q(\mathbf{t},\tau,\mathbf{p}) \le 2\text{mm}$ ensures the maximum tolerance will not exceed

2mm. The constraints $\mathbf{g}(\mathbf{p}) \leq \mathbf{0}$ are geometrical constraints on fixture locations, and are imposed by geometries of parts; $\mathbf{g}(\mathbf{t}, \tau) \leq 0$ are inequality constraints representing lower and upper design bounds for $\mathbf{t}$ and τ.

NCGA is applied to solve Problem (4.3) for the rigid multistation assembly system. The number of function evaluations is $10,000$ at the outer loop. The simulation results and Pareto points are depicted in Figure 4.2 and Figure 4.3 for the optimization process, with and without nested optimization strategy, respectively. As observed from Figure 4.3, the results obtained without applying nested optimization strategy are scattered and unable to provide enough information about the Pareto set. The results in Figure 4.2, obtained using nested optimization strategy, show the tradeoff between cost and SI. The nested optimization strategy has increased the accuracy of the results.

For the rigid assembly system, the runtime is 30 times longer for the process that has nested optimization strategy, than for the process that does not have the strategy, for the same number of function evaluations at the outer loop. This is because in nested optimization strategy, each function evaluation at the outer loop requires an inner optimization process for the tolerance allocation problem. For the rigid assembly system, the time for manufacturing simulation is much less than it is for the inner optimization process, so it takes much less time to evaluate functions that do not have the inner optimization processes. It might be possible to complete the optimization process without nested optimization strategy for 30 times more function evaluations. This would mean that in the above example, $300,000$ function evaluations could be performed. But the results would cause some difficulty in data analysis performed using commercial softwares.

For the compliant assembly system, the time for the inner optimization process

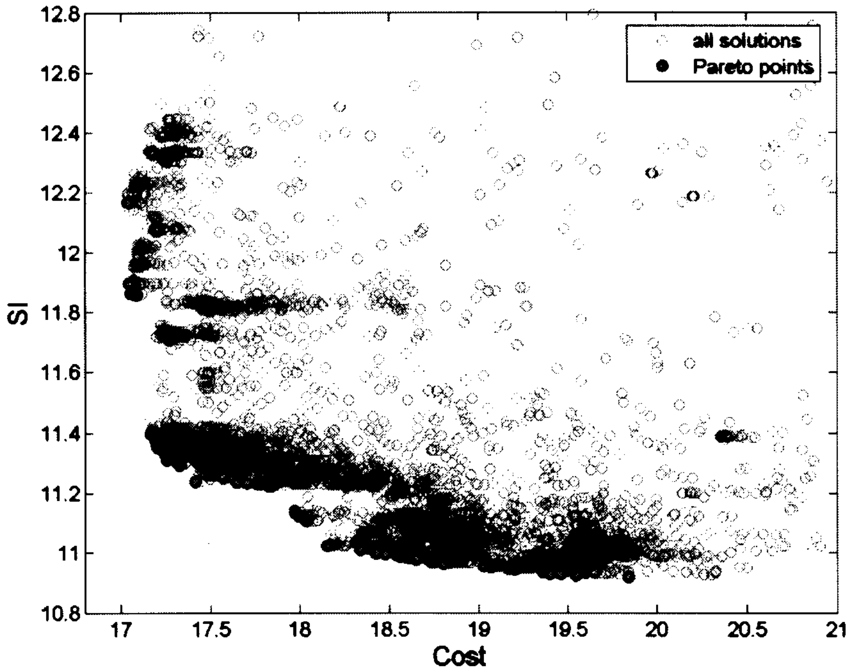

Figure 4.2: Relation between cost and SI for the rigid system using a nested optimization strategy

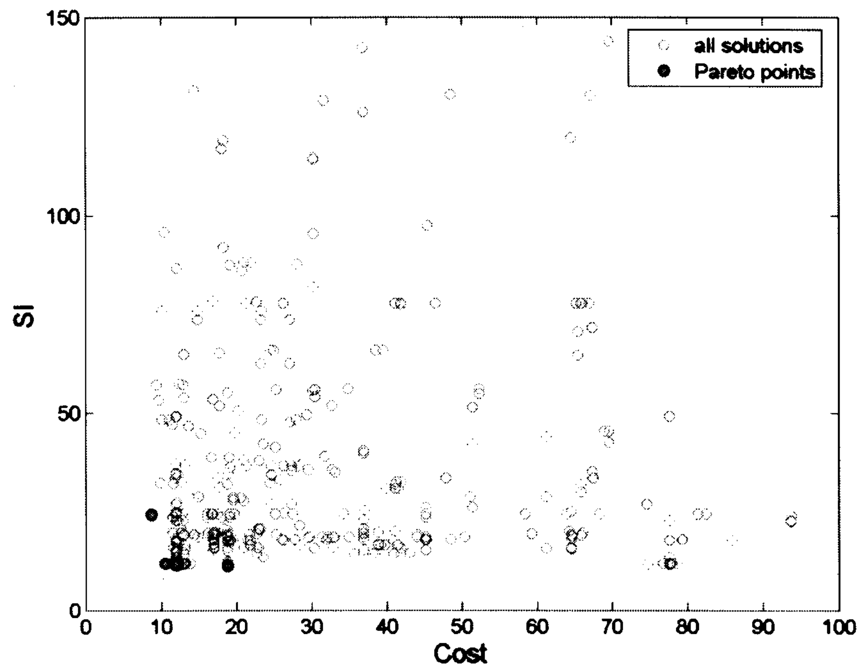

Figure 4.3: Relation between cost and SI for the rigid system without using a nested optimization strategy

can be ignored compared to the manufacturing simulation time. For $10,000$ function evaluations at the outer loop, the same amount of time is required to complete the optimization process with as is required without the nested optimization strategy. In this way, the nested optimization strategy is necessary to increase the efficiency. In summary, the nested optimization strategy has reduced the computational time and increased the accuracy of the results.

4.2.3 Results

The thought that a system with the smallest sensitivity index could incur minimum manufacturing cost can be presented as follows. For two systems, $SI_1 < SI_2$ means that system 1 is more robust than system 2. This can be mathematically expressed as,

$$\begin{aligned} SI_1 < SI_2 \;&\Rightarrow\; \frac{T}{T_1} < \frac{T}{T_2} \\ &\Rightarrow\; T_1 > T_2, \\ &\Rightarrow\; c_1 < c_2, \text{ if } c_i = \frac{1}{T_i}, \end{aligned}$$

where SI is sensitivity index, T is the system output tolerance, and T_i is the system input tolerance. c is the manufacturing cost and it is a reciprocal function of T_i. The concept can also be shown as in Figure 4.4.

The above deduction, however, ignores the fact that in a multistation assembly system, the system input is a vector instead of a single variable. Additionally, the system attributes are defined as a certain norm of the system output vector. The above deduction may not be appropriate for explaining the relations between cost and sensitivity index. Therefore, in this work, the cost-SI relation will be reexamined.

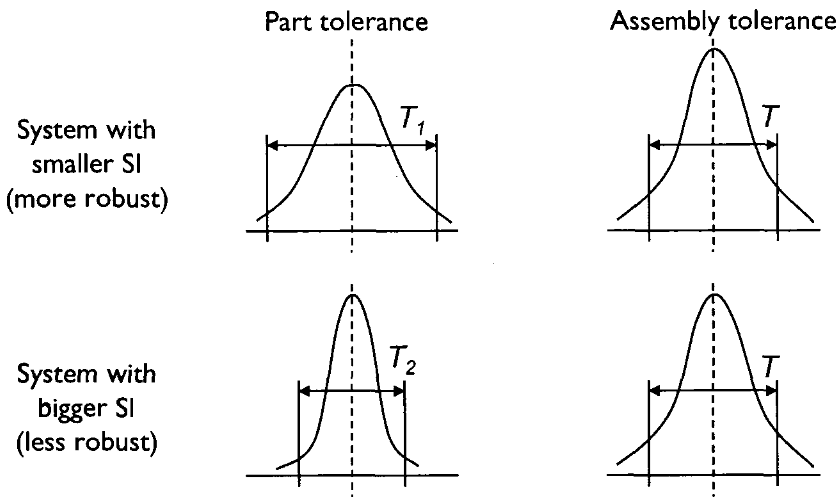

Figure 4.4: Traditional thought about cost and robustness

Tradeoff Between Cost and Sensitivity Index

Problem (4.1) is solved for quality requirement $q_s = 2$mm using nested optimization strategy. Neighborhood Cultivation Genetic Algorithm (NCGA) is applied for 10,000 function evaluations. Figure 4.5 illustrates the cost-SI relation for the rigid multistation assembly example as shown in Figure 3.1. The observation shows that there are tradeoffs between the system cost and the sensitivity index. The 12-percent $(\frac{12.2-10.9}{10.9})$ decrease in the sensitivity index can result in an 18-percent increase in system cost.

Problem (4.1) is also solved for a compliant multistation assembly system at the quality requirement $q_s = 2$mm, using nested optimization strategy. NCGA is applied for 10,000 function evaluations. Figure 4.6 illustrates the cost-SI tradeoffs for the compliant multistation assembly example of Figure 3.3. The three percent decrease in the sensitivity index can result in a seven percent increase in system cost.

The tradeoff between the system cost and the sensitivity index does not exist for only $q_s = 2$mm. Problem (4.1) is solved for different quality requirements at

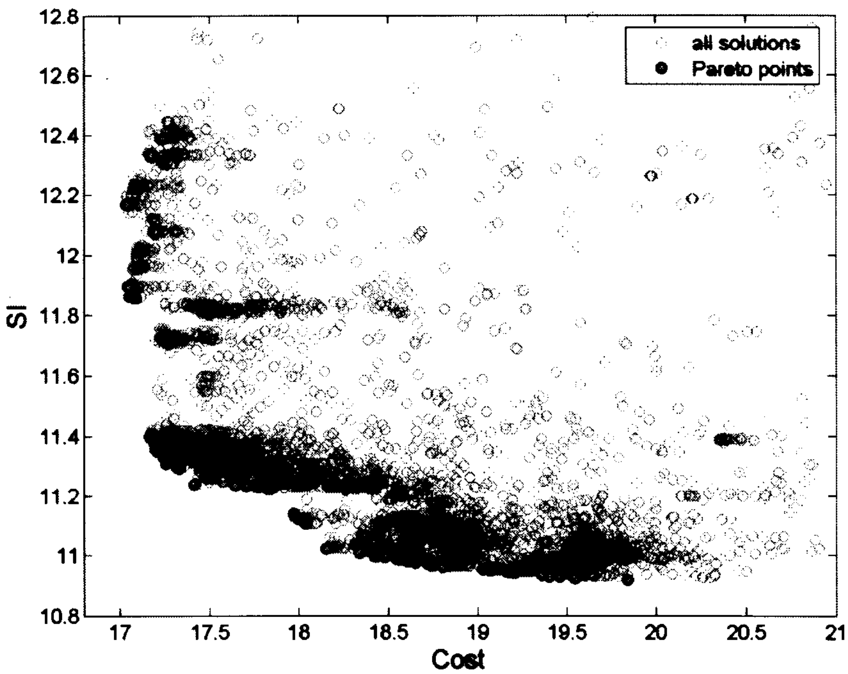

Figure 4.5: Relation between cost and SI for the rigid system ($q_s = 2$mm)

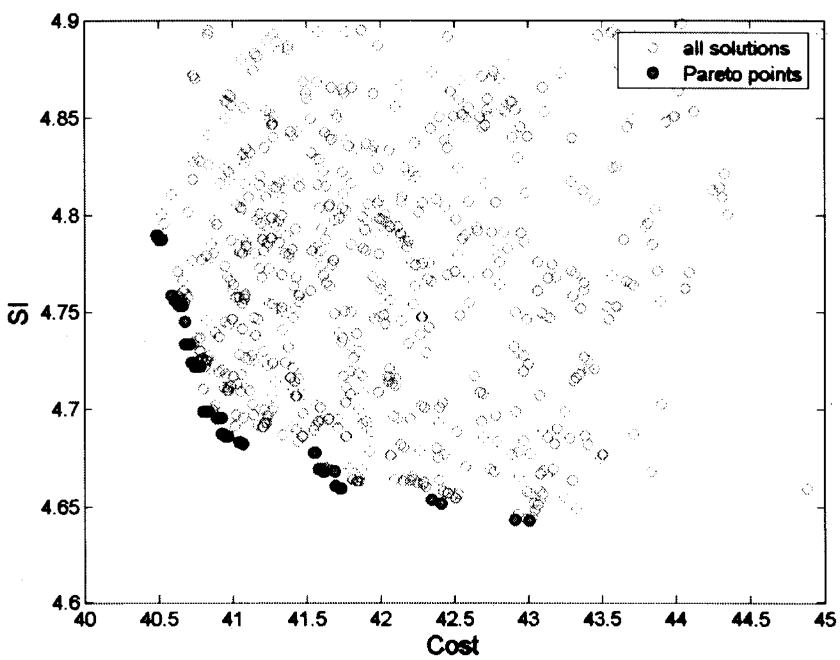

Figure 4.6: Relation between cost and SI for the compliant system ($q_s = 2$mm)

q_s = 1.3mm, 1.5mm, 1.8 mm, and 2mm for the rigid multistation assembly example. The results are seen in Figure 4.7. Again, there are tradeoffs for the system cost and the sensitivity index.

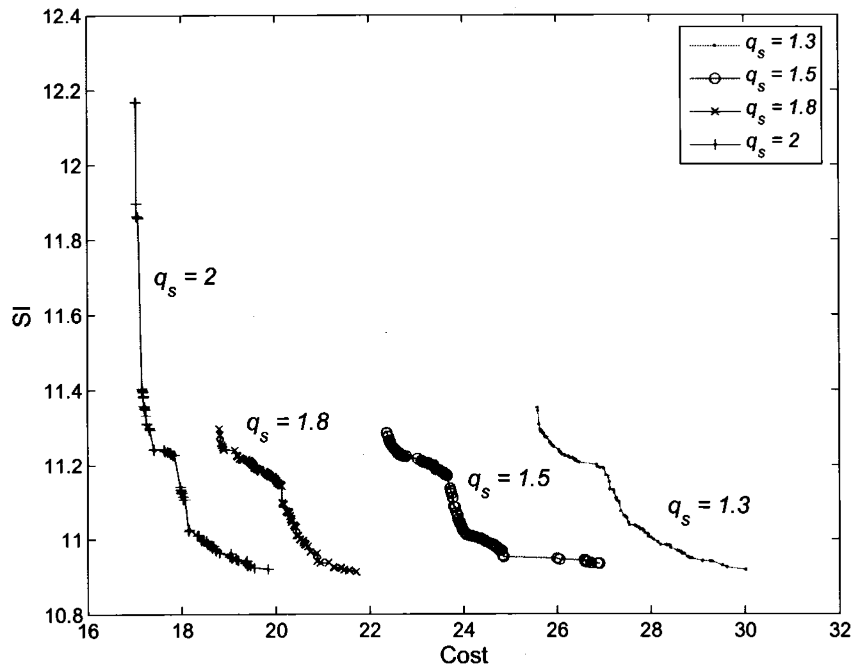

Figure 4.7: Relation between cost and SI for the rigid system (q_s = 1.3mm to 2mm)

In order to explain the tradeoff, the definitions for sensitivity index SI, and the cost-tolerance relations are presented below.

For a rigid multistation assembly system, the sensitivity index SI is defined as,

$$SI \equiv S_{max} \equiv \sup_{\mathbf{u}\neq 0} \frac{\mathbf{u}^T\mathbf{D}^T\mathbf{D}\mathbf{u}}{\mathbf{u}^T\mathbf{u}} = \|\mathbf{D}\|_2^2 = \lambda_{max}(\mathbf{D}^T\mathbf{D}), \tag{4.4}$$

where $\mathbf{D}$ is derived by reformulating the multistation variation propagation model and eliminating all intermediate state variables (please refer to page 68). The fixture design information is modeled by $\mathbf{D}$, which is independent of variation input (represented by $\mathbf{u}$).

For a compliant multistation assembly system, the sensitivity index SI can be defined as the norm of a tolerance output vector $\mathbf{t}_0$ with respect to the norm of unit product and process tolerance input vectors:

$$
\begin{aligned}
SI &= \|\mathbf{t}_0\|, \qquad (4.5)\\
\mathbf{t}_k &= \mathbf{F}_{t_k}(\mathbf{t}_{k+1}, \tau_k),\ k \in 0, 1, 2, \ldots, N-1,\\
\hat{\mathbf{t}_N} &= \mathbf{1},\ \text{and}\\
\hat{\tau_k} &= \frac{\mathbf{1}}{\mathbf{3}},\ k \in 1, 2, \ldots, N-1.
\end{aligned}
$$

The process tolerance value is set as one-third of the product tolerance value. The sensitivity matrices in tolerance transfer model $\mathbf{F}_{t_k}$ depend on fixture positions $\mathbf{p}$ and are obtained using compliant assembly variation analysis (CAVA) with finite element models.

Beside the above two definitions for a sensitivity index, there are other choices of robustness criteria in optimal fixture layout design. For example, for rigid multistation assembly systems, the frequently used criteria include D-optimality (minimize $\det(\mathbf{D}^T\mathbf{D})$), A-optimality (minimize $\mathrm{tr}(\mathbf{D}^T\mathbf{D})$), and E-optimality (minimize the extreme eigenvalue of $(\mathbf{D}^T\mathbf{D})$), where $\mathrm{tr}(\cdot)$ and $\det(\cdot)$ are the trace and the determinant of a matrix, respectively. Each robustness criteria (sensitivity index) reflects only one aspect of the characteristics of the sensitivities to the final product attributes. For example, the A-optimality focuses on the sum of the sensitivities, while the E-optimality focuses on the maximum or minimum sensitivity.

In summary, the robustness depends on fixture positions $\mathbf{p}$. The changes in $\mathbf{p}$ will modify the contributions of parts and tooling errors (tolerances $\mathbf{t}$) on the final assembly variation.

On the other hand, for cost-tolerance relations, exponential and reciprocal functions are used for their decent data fit and simple function structures. In this work, the exponential function is chosen to represent the cost of the incoming parts of the compliant multistation assembly system,

$$c(t) = e^{-3t}, \tag{4.6}$$

where t is the product or process tolerance design variable.

For a tolerance design vector, the cost-tolerance model is

$$c(\mathbf{t}) = \sum_{i=1}^{n} e^{-3t_i}, \tag{4.7}$$

where n is the size of the tolerance vector.

For the rigid multistation assembly system, the reciprocal function is chosen as follows:

$$c(t) = \frac{1}{t}. \tag{4.8}$$

For a tolerance design vector, the cost-tolerance model is

$$c(\mathbf{t}) = \sum_{i=1}^{n} \frac{1}{t_i}, \tag{4.9}$$

where n is the size of the tolerance vector.

Although the sensitivity index and the system cost depend on different design variables, fixture locations $\mathbf{p}$ or tolerance vectors $\mathbf{t}$, the functions both describe the contributions of parts and tooling errors (tolerances) to the system attributes. The product and process tolerances contribute equally to the system cost, according to the cost-tolerance relations. Their contributions to the sensitivity index essentially depend on the variation propagation model at the current fixture layout.

When the system cost and the sensitivity index are the objectives of the two-objective optimization problem, the optimal solutions are sought for a balance between the different contributions to the objectives. Finally, the difference in the contributions results in the tradeoff between the system cost and the sensitivity index.

Another design problem is solved to provide further support to the above analysis. A-optimality is used for the fixture layout design instead of E-optimality. The problem is formulated as

$$\begin{aligned} \min_{\mathbf{t},\tau,\mathbf{p}} \quad & \{c(\mathbf{t},\tau), SI(\mathbf{p})\} \\ \text{s.t.} \quad & q(\mathbf{t},\tau,\mathbf{p}) \le q_s \\ & \mathbf{g}(\mathbf{t},\tau) \le 0 \\ & \mathbf{g}(\mathbf{p}) \le \mathbf{0}, \end{aligned} \tag{4.10}$$

where

$$SI = \mathrm{tr}(\mathbf{D}^T\mathbf{D}) = \sum_{i=1}^{p} \lambda_i, \tag{4.11}$$

and the design variables $\mathbf{t}$ and τ are product and process tolerance design vectors, respectively. The design variables $\mathbf{p}$ are the fixture locations. Again, the nested optimization strategy is applied, and NCGA is used for $10,000$ function evaluations at the outer loop. The problem is solved for the quality requirements $q_s = 1.5$mm and $q_s = 2$mm. The results are shown in Figure 4.8.

The observation shows the tradeoff between the system cost and the sensitivity index. A 14-percent decrease in the sensitivity index can result in a 20-percent increase in the system cost. The percentages are similar with the case of E-optimality, where a 12-percent decrease in the sensitivity index can result in an 18-percent increase in the system cost.

From Chapter III, it is known that the lowest sensitivity index for the rigid multistation assembly system is 10.840 using the E-optimality. Another observation from Figure 4.7 is that the sensitivity indices at the Pareto solutions are in the

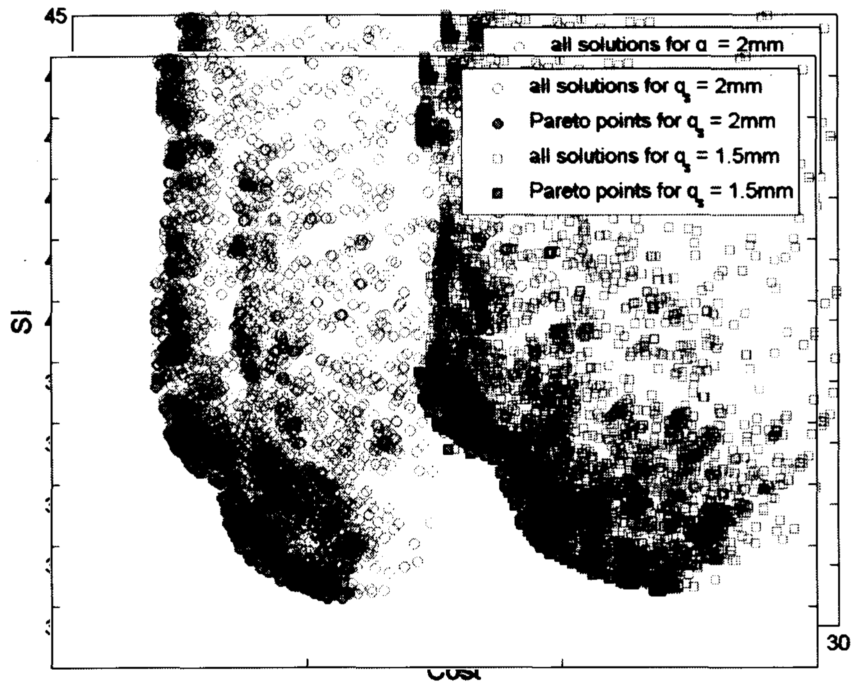

Figure 4.8: Relation between cost and SI(trace) for the rigid system (q_s = 1.5mm and 2mm)

range between 10.84 and 11.50. Therefore, the tradeoffs exist for fixture layouts with sensitivity indices that are close to the lowest sensitivity index.

Critical Quality Requirement

The tradeoff between the system cost and the sensitivity index does not exist for all quality requirements. For the rigid multistation assembly system, Problem (4.1) is then solved to address the effects of q_s on cost-SI tradeoff relations, for various quality requirements $q_s = 3, 4, \ldots, 10$mm. The results can be seen in Figure 4.9. The observation shows that the tradeoff becomes less and less significant with increasing quality requirement values. At the quality requirement $q_s = 10$mm, there is only one solution and no tradeoff.

To explain this result, five Pareto solutions (see Table 4.2) are selected from

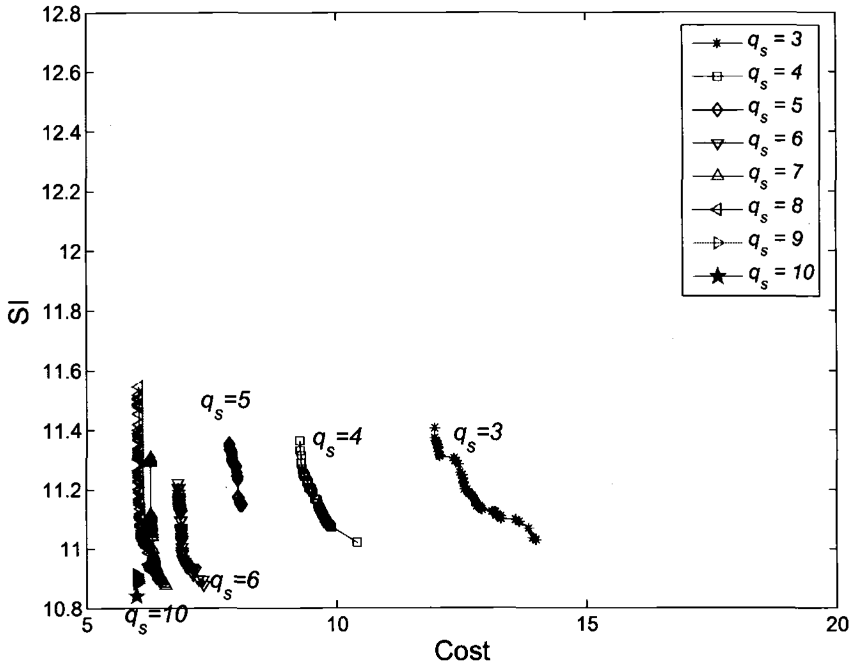

Figure 4.9: Relation between cost and SI for the rigid system (q_s = 3mm to 10mm)

Table 4.2: Selected fixture layout designs for the rigid multistation assembly system

Fixture Positions **p**	Layout 1	Layout 2	Layout 3	Layout 4	Layout 5
P1 x	352.69	352.39	352.5	329.79	351.66
P1 z	350.68	320.39	320.39	320.33	320.33
P2 x	339.67	340.39	340.39	340.39	340.39
P2 z	43.41	38.152	38.152	37.61	38.181
P3 x	837.62	837.62	837.45	837.45	931.99
P3 z	614.93	614.93	613.75	613.75	614.93
P4 x	1534.5	1534.4	1538.9	1534.4	1534.4
P4 z	70.022	36.977	294.8	336.57	623.04
P5 x	2236.3	2148.7	1726.6	1726.5	1730.3
P5 z	88.322	83.083	83.083	281.92	285.9
P6 x	2253	2252.8	2327.1	2324.7	2324.4
P6 z	546.99	546.99	546.97	546.97	546.97
P7 x	2741	2741	2861.5	2861.5	2861.5
P7 z	74.456	74.456	74.452	74.452	74.527
P8 x	2697	2697	2805.1	2805.4	2691.1
P8 z	383.58	443.45	469.7	443.11	469.7
SI	11.929	11.770	11.221	11.186	11.167
Cost	17.484	18.086	19.084	19.709	20.905

Figure 4.9 at $q_s = 2$mm. The tolerance allocation Problem (2.13) is then solved to show the quality-cost tradeoffs at each fixture layout. The results are depicted in Figure 4.10.

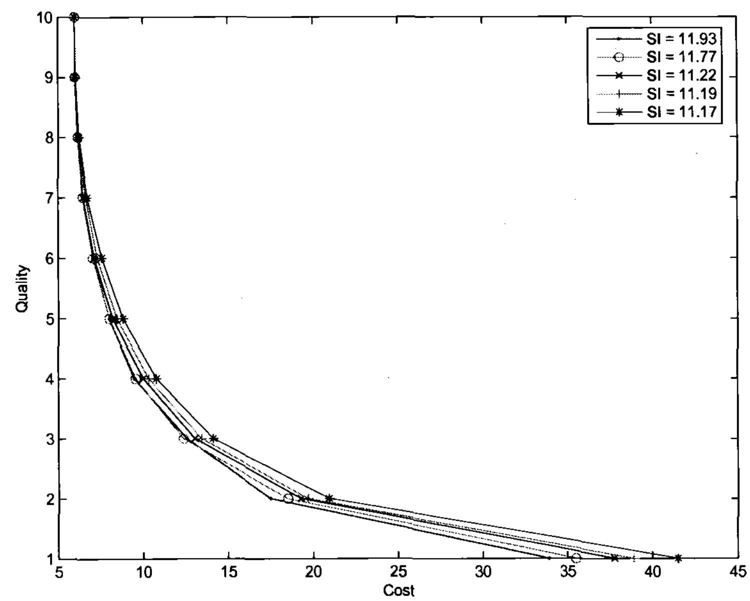

Figure 4.10: Cost-quality tradeoffs for selected fixture layout designs in Table 4.2

At $q_s = 10$mm, all 12 of the tolerance design variables reach the upper bound of 2mm for selected fixture layouts. Therefore, the minimum cost for the system is $c = 12 \times \frac{1}{t} = 12 \times \frac{1}{2(\mathrm{mm})} = 6$. With the same cost value, the two-objective problem becomes a single objective problem, sharing the same problem formulation with Problem (3.1). Then the goal is to find a fixture layout that provides the minimum sensitivity index. So, at $q_s = 10$mm, there is only one solution that ensures both the minimum system cost and the minimum sensitivity index.

Based on this analysis, one concept is introduced as the critical quality requirement. The critical quality requirement q_c is defined as the final product quality evaluated at the optimal fixture layout, with all the tolerance variables at their upper design bounds. Problem (4.1), solved at $q_s = q_c$, has only one solution. At this solution, all tolerances reach the upper design bounds. According to the cost-tolerance relations, the cost is the minimum for the assembly system. Additionally, the sensitivity index is the lowest for the assembly system, and is equal to the solution, as seen by solving Problem (3.1) in Chapter III.

For the assembly system design, if the quality requirement is less than the critical quality requirement, a choice must be made between cost and SI along the Pareto curve. Otherwise, the solution at the critical quality requirement should be chosen, ensuring both the minimum cost and the minimum SI. Then the design goal for the multistation assembly system becomes to decrease the critical quality requirement.

In the example, the critical quality requirement q_c is far from the expected product quality, which is always less than 2mm. There are two ways to decrease q_c for an assembly system. One way is to change station characteristics, such as the assembly sequence. The other way is to decrease the upper design bounds, for both product and process tolerance variables. The Pareto frontier is a mapping from the design space, such that the design bound selection for input tolerances is important to system attributes. For example, when the upper bound of the tolerance design is changed from 2mm to 1.5mm, the critical quality requirement q_c changes from approximately 10mm to 7mm (see Figure 4.11).

4.3 Relations Between Quality and Robustness

After studying the relations between system cost and system robustness, it is also interesting to explore the relation between final product quality and system robustness. For final product quality and system robustness, both depend on the variation propagation models, so the contributions of parts and tooling errors to the quality and system robustness are similar. The observed tradeoff between the quality

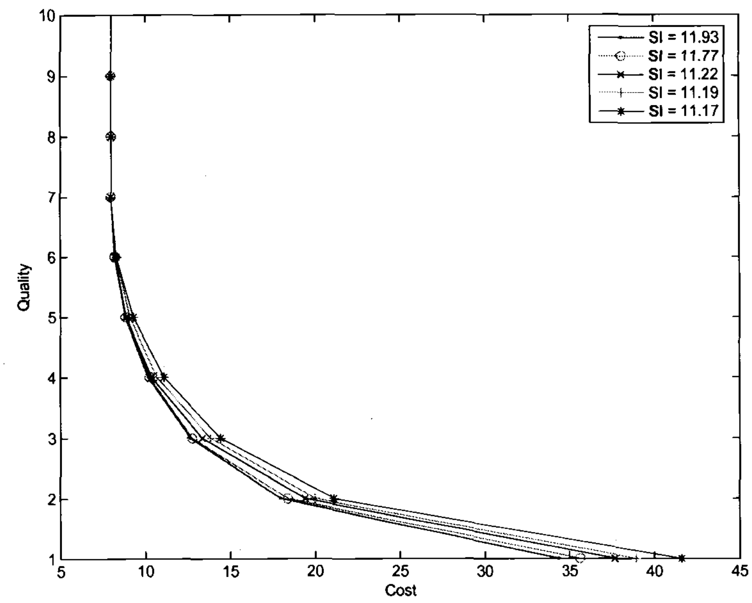

Figure 4.11: Cost-quality tradeoffs for selected fixture layout designs, with upper design bound of 1.5mm for tolerance variables

and robustness deserves further discussion.

4.3.1 Problem Formulation

The relations between final product quality and system robustness are studied by solving a two-objective optimization problem. The problem is formulated as follows based on the quality-driven tolerance allocation problem (2.17) and the optimal fixture layout design problem (3.1):

$$\begin{aligned} \min_{\mathbf{t},\tau,\mathbf{p}} \quad & \{q(\mathbf{t},\tau,\mathbf{p}), SI(\mathbf{p})\} \\ \text{s.t.} \quad & c(\mathbf{t},\tau) \leq b \\ & \mathbf{g}(\mathbf{t},\tau) \leq 0 \\ & \mathbf{g}(\mathbf{p}) \leq \mathbf{0}, \end{aligned} \tag{4.12}$$

where the design variables $\mathbf{t}$ and τ are product and process tolerance design vectors, respectively. The design variables $\mathbf{p}$ are the fixture locations.

q is the final product quality, which is dependent on all design variables. SI is the sensitivity index, which can be evaluated by variation propagation models, given fixture locations $\mathbf{p}$. c is the system cost, which is a function of $\mathbf{t}$ and τ.

The constraints $\mathbf{g}(\mathbf{p}) \leq \mathbf{0}$ are geometrical constraints on fixture locations, and are imposed by geometries of parts. $\mathbf{g}(\mathbf{t}, \tau) \leq 0$ are inequality constraints representing lower and upper design bounds for $\mathbf{t}$ and τ. b is the budget. The constraint $c(\mathbf{t}, \tau) \leq b$ ensures the system cost will not exceed the budget.

4.3.2 Nested Optimization Strategy

Using the nested optimization strategy, Problem (4.12) can also be written as

$$\begin{aligned} \min_{\mathbf{p}} \quad & \{\min_{\mathbf{t},\tau}\{q(\mathbf{t}, \tau, \mathbf{p}) | c(\mathbf{t}, \tau) \leq b, \mathbf{g}(\mathbf{t}, \tau) \leq 0\}, SI(\mathbf{p})\} \\ \text{s.t.} \quad & \mathbf{g}(\mathbf{p}) \leq \mathbf{0}. \end{aligned} \tag{4.13}$$

As shown in Figure 4.12, the *outer loop* is used to determine values for fixture layout design variables $\mathbf{p}$. The sensitivity matrices are obtained by applying manufacturing models, given fixture locations $\mathbf{p}$.

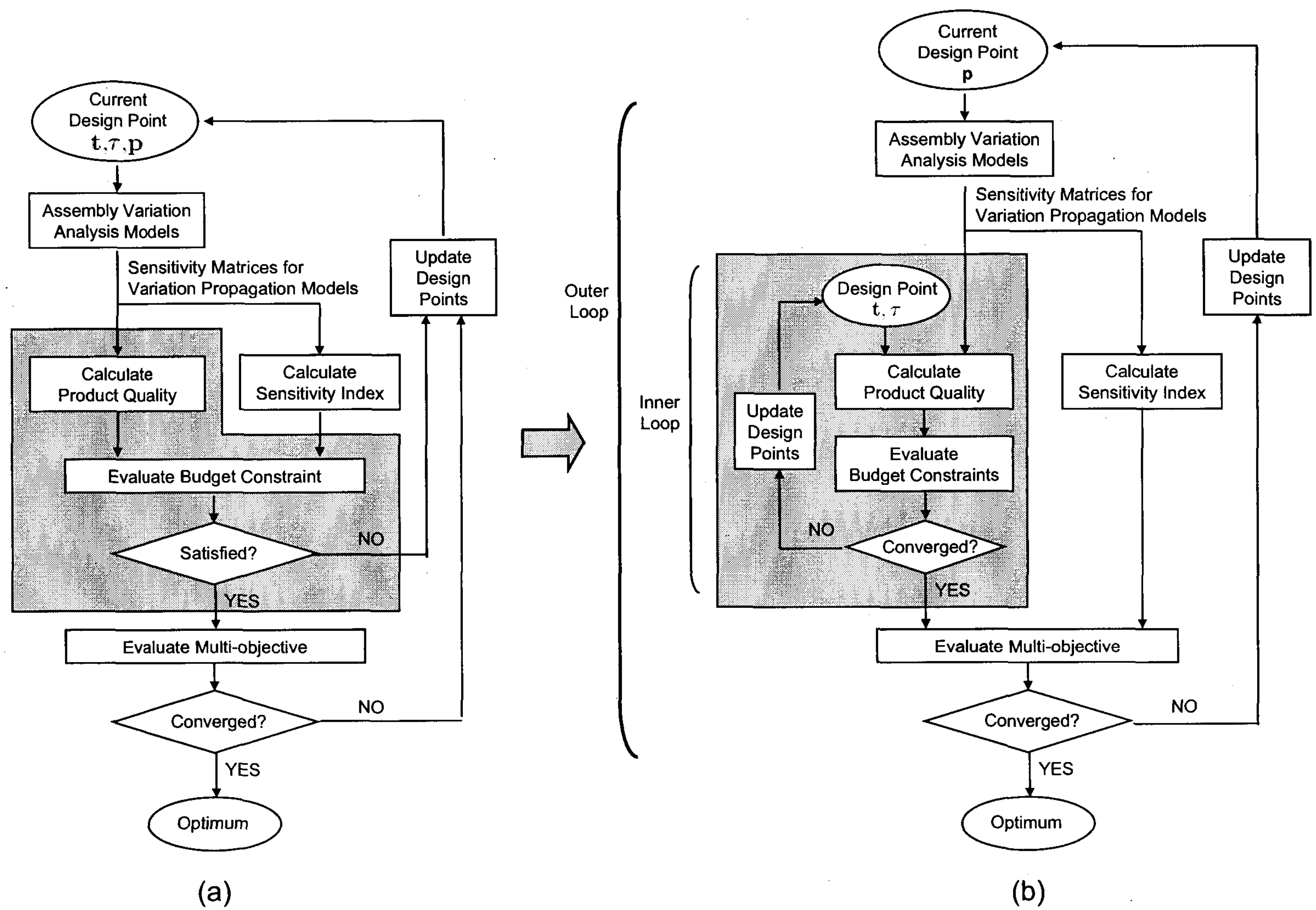

Figure 4.12: Nested optimization strategy for Problem (4.12)

These sensitivity matrices are used as parameters in variation propagation models for the inner loop optimization process. For the *inner loop*, given the variation propagation models and cost models, product and process tolerances, $\mathbf{t}$ and $\boldsymbol{\tau}$, will be allocated to provide the best product quality q within the budget b.

Problem (4.12) and Problem (4.13) are equivalent for evaluating quality-SI relations because the two problem formulations can provide the same objective value for any feasible design point $\mathbf{p}$. The proof is similar to the proof for Lemma on page ??.

Gradient-based optimization algorithms can be selected to solve the tolerance allocation problem in the inner loop. Derivative-free optimization algorithms, such as NCGA, should be used for the outer loop.

4.3.3 Results

Tradeoff Between the Final Product Quality and Sensitivity Index

Problem (4.12) is then solved for budgets of $b = 7, 10, 15, 20, 25, 30, 35,$ and 40, for the rigid multistation assembly example as shown in Figure 3.1. Nested optimization strategy and NCGA are used for 10,000 function evaluations. The results can be seen in Figure 4.13. The observation shows that there are tradeoffs between the final product quality and the sensitivity index. The five percent decrease in the sensitivity index can result in an eight to 13-percent increase in the final product quality.

In order to explain the tradeoffs between the final product quality and the sensitivity index, the definition for final product quality is reviewed. With given tolerance information, the final product quality can be defined as $q = \|\mathbf{t}_{01}\|_\infty$, where $\mathbf{t}_{01}$ is the product tolerance vector of the final assembly.

Unlike the analysis for the cost-SI tradeoff, the final product quality and SI both depend on the variation propagation models, so the contributions of parts and tooling errors to the final product quality and SI are similar. Noting that the budget

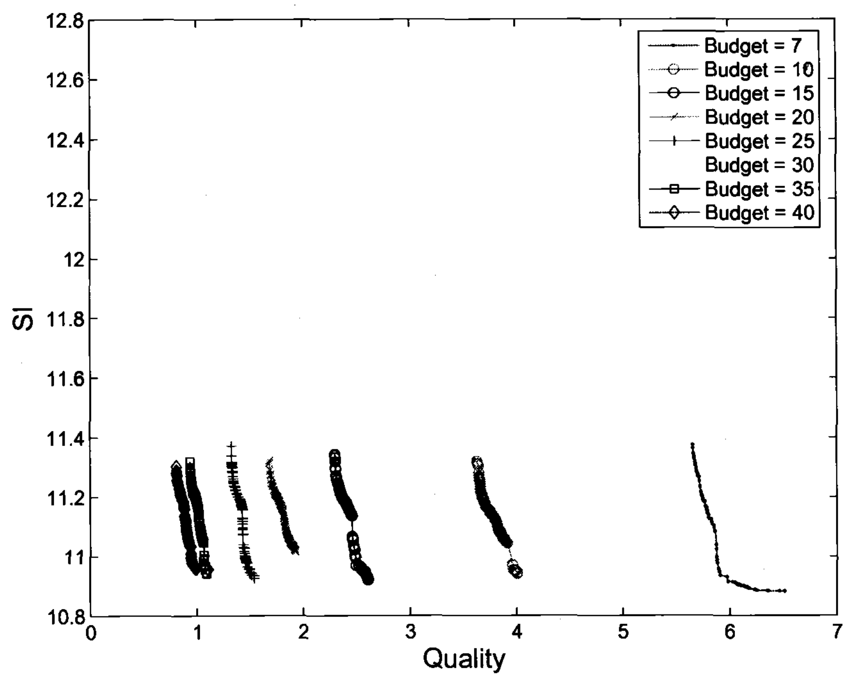

Figure 4.13: Relation between quality and SI for the rigid system

constraint is active in Problem (4.12), the influence of the contribution from tolerance variables to the system cost cannot be ignored. As presented in Section 4.2.3, the product and process tolerances contribute equally to the total system cost, according to cost-tolerance relations. The difference of these two kinds of contributions resulted in the tradeoff between the system cost and the sensitivity index, and also results in the tradeoff between the final product quality and the sensitivity index.

Critical Budget Requirement

Another observation from Figure 4.13 is that the tradeoff between the final product quality and the sensitivity index becomes less and less significant as budgets increase. A critical budget requirement b_c is then defined as the cost calculated at the optimal fixture layout, with all tolerance variables at their lower design bounds.

For the assembly system design, if the budget is less than the critical budget requirement, a choice has to be made between quality and SI along the Pareto curve. Otherwise, Problem (4.12) solved at $b = b_c$ has only one solution. The solution at the critical quality requirement should be chosen, ensuring both the best quality and the minimum SI.

Then the design goal for the multistation assembly system becomes to decrease the critical budget requirement. This can be realized by changing the station characteristics or increasing the lower design bounds, for both product and process tolerance variables.

4.4 Relations Among Cost, Quality, and Robustness

The relations among cost, quality and robustness are studied by solving a three-objective optimization problem. The mathematical formulation is

$$\begin{aligned} \min_{\mathbf{t},\tau,\mathbf{p}} \quad & \{c(\mathbf{t},\tau), q(\mathbf{t},\tau,\mathbf{p}), SI(\mathbf{p})\} \\ \text{s.t.} \quad & \mathbf{g}(\mathbf{t},\tau) \leq 0 \\ & \mathbf{g}(\mathbf{p}) \leq \mathbf{0}, \end{aligned} \tag{4.14}$$

where the design variables $\mathbf{t}$ and τ are product and process tolerance design vectors, respectively. The design variables $\mathbf{p}$ are the fixture locations; c is the system cost in the assembly processes, depending only on $\mathbf{t}$ and τ; SI is the sensitivity index, which can be evaluated by variation propagation models, given fixture locations $\mathbf{p}$; and q is the final product quality.

The constraints $\mathbf{g}(\mathbf{p}) \leq \mathbf{0}$ are geometrical constraints on fixture locations, imposed by geometries of parts; $\mathbf{g}(\mathbf{t},\tau) \leq 0$ are inequality constraints that represent lower and upper design bounds for $\mathbf{t}$ and τ. There is no quality or budget requirement. NCGA is applied for 40,000 function evaluations. The results can be seen in

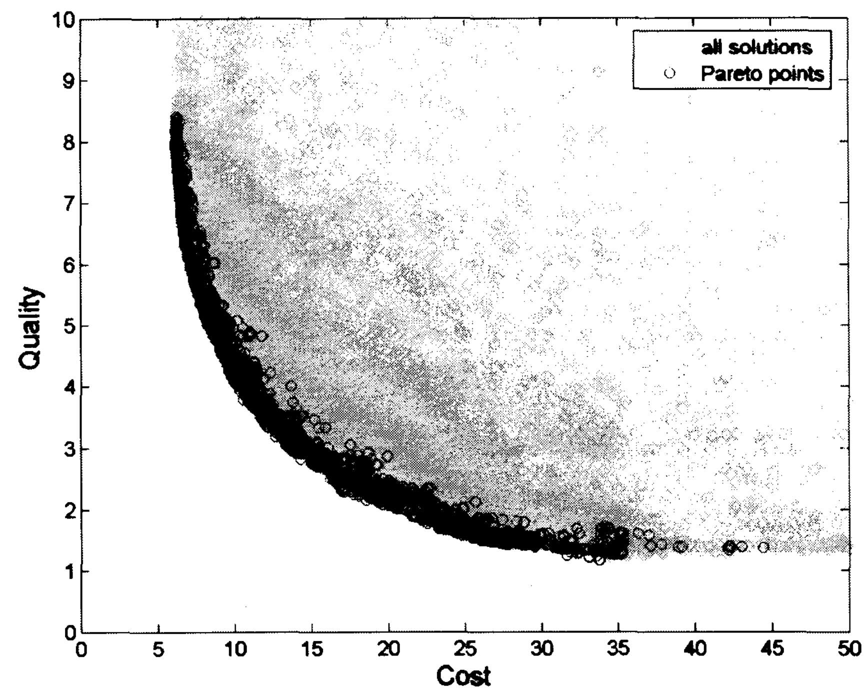

Figure 4.14: Relations between cost and quality for the rigid system

Figure 4.14, Figure 4.15, and Figure 4.16.

Figure 4.14 shows that there are tradeoffs between the final product quality and the system cost. There is no tradeoff for cost and SI, and for quality and SI, according to Figure 4.15 and Figure 4.16. The cost-SI tradeoff exists only when there is a quality requirement, and the constraint is active (the product quality being equal to the quality requirement). The quality-SI tradeoff exists only when there is a budget requirement, and the constraint is active (the cost being equal to the budget).

4.5 Summary

A framework was proposed and presented to consider tolerance allocation and fixture layout design simultaneously. Nested optimization strategy was proposed to solve the multiobjective problems. The tradeoff between cost and robustness, and the

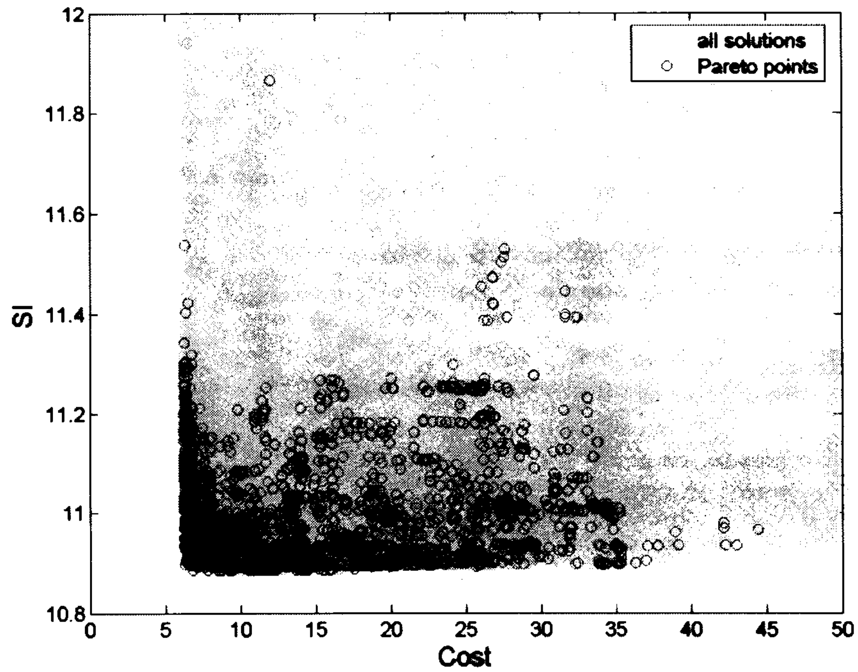

Figure 4.15: Relations between cost and SI for the rigid system

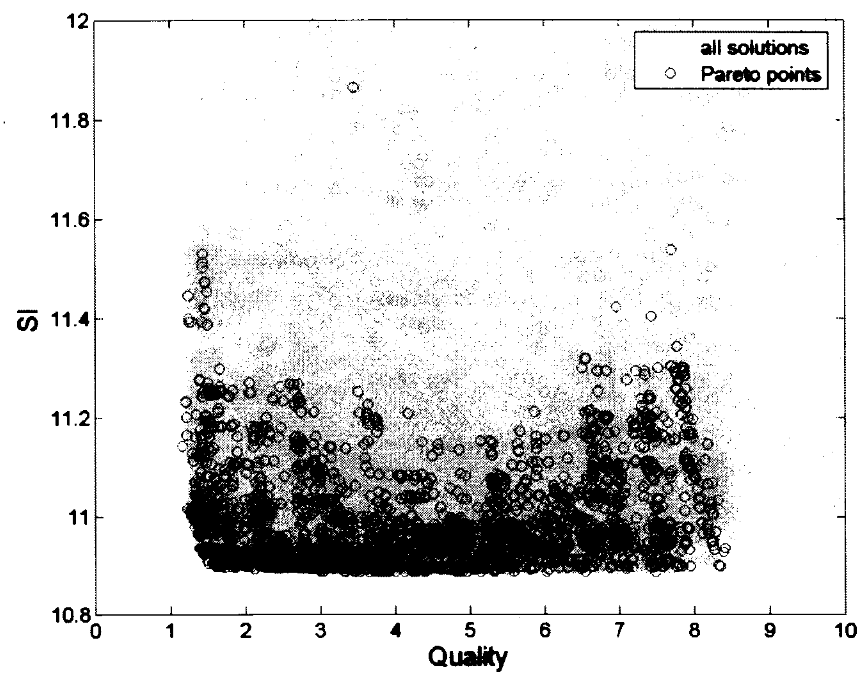

Figure 4.16: Relations between quality and SI for the rigid system

tradeoff between quality and robustness were demonstrated for rigid and compliant multistation assembly examples. The existence of tradeoffs makes it necessary to define criteria that can properly assess the design of multistation assembly systems. The critical quality and budget requirements were defined in this dissertation. By changing the design bounds of the product and process tolerance variables, it is possible to find a design that results in the best product quality, the lowest system cost, and the greatest system robustness.

CHAPTER V

Conclusions

This dissertation has proposed and demonstrated methodologies for tolerance allocation, fixture layout optimization, and the integration of tolerance allocation and fixture layout design, in multistation assembly systems. System cost, final product quality, and system robustness are three attributes of major importance considered in the systems. Accordingly, single objective and multiobjective problems are formulated and solved for design decisions on product tolerances, process tolerances, and fixture locating positions.

There has been extensive research activity in tolerance analysis and allocation, but limited work has been done for compliant assembly systems, even though compliant assemblies are widely used in manufacturing industries. Tolerance analysis models, cost-tolerance relations, and appropriate optimization algorithms are necessary during the process of allocating tolerances. Accordingly, in Chapter II multistation variation propagation models, tolerance-variation models, and cost-tolerance relations were integrated for compliant multistation assembly systems. Based on manufacturing models and cost-quality tradeoffs, quality-driven tolerance allocation problems were formulated with respect to minimizing variations of final product dimensions propagated from incoming part variations and fixture variations, while meeting the budget requirements. Cost-driven tolerance allocation problems were formulated with respect to minimizing costs associated with product and process tolerances, while satisfying quality requirements.

The optimal results showed that the Pareto set representing the tradeoff between cost and quality can be generated by solving either the cost-driven tolerance allocation problem or the quality-driven tolerance allocation problem. In quality-driven tolerance allocation, different parts may contribute differently to final product quality. Tolerances and purchasing budgets for incoming parts should be allocated accordingly. The cost-driven tolerance allocation demonstrated that variations due to fixtures cannot be ignored in the tolerance allocation process, which requires an increased budget or better quality for sensitive parts. Additionally, the consideration of process variations results in different tolerance allocation schemes for products.

Methodologies were proposed and applied to solve these tolerance allocation problems using all-in-one (AIO) and analytical target cascading (ATC) strategies. Tolerance allocation can be modeled as a hierarchical multilevel optimization problem and the feasibility of the ATC strategy was demonstrated on a compliant multistation assembly example. For the small scale problem of the compliant assembly example, the ATC approach highlights its advantage over AIO by being able to offer intermediate specifications without additional analysis. It is emphasized that the ATC strategy is particularly advantageous when considering large-scale and complex multistation assembly design problems that are difficult to solve using the AIO approach. Additionally, in ATC, after appropriate specifications have been obtained for each subproblem (assembly station), it is not necessary to resolve the entire ATC problem when some characteristics or parameters change at a particular station; only the design subproblem associated with that particular station must be resolved as long as the ATC-obtained specifications are satisfied.

Dimensional quality of final products depends on both the input variation level and the process sensitivity to variation inputs. The former issue can be addressed by tolerance allocation design. The latter can be addressed by optimal design of fixture layouts in a multistation assembly process so that the process is insensitive to input variations. Therefore, in Chapter III, the fixture layouts were optimized to improve the system robustness with a compliant multistation assembly example. The methods reviewed in rigid systems are not applicable to compliant systems, because the involvement of finite element models results in high computational cost, and requires the integration of optimization algorithms with finite element analysis.

Sensitivity index, a quantitative measure of system robustness, was defined to evaluate different fixture layout designs based on compliant multistation variation propagation models. Grid IDs were used as position design variables to avoid expensive simulations from multipoint constraints and remeshing algorithms. Accordingly, the mixed-discrete derivative-free optimization algorithms DIRECT and MIGA were chosen to solve the problems. The applications showed that a sensitivity index definition, a selection of design variables for fixture locations, and an appropriate optimization algorithm enabling the integration with finite element analysis tools, are the keys for successful fixture layout design in compliant assembly systems. The optimal results of fixture layout design addressed the variation propagation and interactions among stations to improve the robustness for multistation assembly systems, and resulted in designs different from the fixture layout design of a single workpiece or a single station.

In Chapter II and Chapter III, tolerance allocation and optimal fixture layout design were formulated as single objective problems and conducted independently for more cost savings, better product quality, or improved system robustness. There is no general framework to analyze their interactions qualitatively or quantitatively, or to study their integrated effects on product quality, cost, and system robustness. In Chapter IV, design possibilities were explored to address the interactions of tolerance allocation and fixture layout design by formulating multiobjective design problems. With increasing problem scale, nested optimization strategies were proposed to solve these problems, integrating them with gradient-based optimization in the inner loop and mixed-discrete optimization in the outer loop. Examples showed that nested optimization strategies provided advantages by shortening computational time and increasing the accuracy of the results.

When tolerance allocation and fixture layout design were considered simultaneously, the examples demonstrated the tradeoffs between cost and robustness, and the tradeoffs between final product quality and robustness. This goes against the traditional thinking that a system with smallest sensitivity index will incur minimum manufacturing cost. The analysis of manufacturing models addressed the fact that different contribution patterns of product and process variations to the system attributes result in these tradeoffs. Accordingly, the critical quality and budget requirements were defined in the dissertation, to serve as evaluation criteria for multistation assembly systems, and to ensure best product quality, system robustness, and minimum cost. It was suggested that when customer requirements or product design goals for quality are known, a cost-SI tradeoff analysis is necessary. When an enterprise budget is assigned to the manufacturing department, a quality-SI tradeoff analysis is required.

5.1 Contributed Methodologies

The general contribution of this dissertation is the analysis and integration of tolerance allocation and fixture layout design for multistation assembly systems. The demonstrated tradeoffs between system attributes prove that the two design activities are not separable and must be considered simultaneously instead of independently or sequentially.

Specific contributions of this dissertation are summarized as follows

- A general design methodology was developed for allocating product and process tolerances in compliant multistation assembly systems, with respect to minimizing manufacturing costs or minimizing variations of final products.
- The applicability of analytical target cascading methodology to tolerance allocation in compliant multistation assembly systems was demonstrated.
- A general design methodology was developed for locating fixtures in compliant multistation assembly systems by minimizing the sensitivity index, which is a quantitative measure of system robustness.
- A general framework for integrating tolerance allocation with fixture layout design in multistation assembly systems, was proposed for demonstrating tradeoffs between cost and robustness, and tradeoffs between quality and robustness.

5.2 Suggestions for Future Work

The following research issues deserve further investigation:

1. Inclusion of tooling variations. In the dissertation, only the variation from fixtures are considered in the tolerance allocation process. Variations due to

assembly tools such as welding guns, and variations due to additional holding fixtures, would also have effects on the final product quality and tolerance allocation scheme of incoming part tolerances. These should be incorporated into the design.

2. Fixture layout design for "N-2-1" locating scheme. This dissertation gave an example of a compliant multistation assembly system with a "3-2-1" locating scheme. Further applications on assembly systems can be conducted, using the "N-2-1" fixture layout design for compliant multistation assembly systems. With increasing number of fixtures and design variables, the efficiency of optimization process needs to be improved further.

3. Construction of utility function. The existence of tradeoffs makes it necessary to derive a utility function for better consideration of multiple objectives (quality, cost, and robustness). The interaction among engineering and other related subjects should be thoroughly examined in order to formulate an effective utility function for decision-making processes in assembly systems.

4. Extension of multilevel design strategy to large-scale manufacturing applications. As manufacturing models increase in complexity, and manufacturing applications include more stations and products, design problems become more difficult to solve. It is then necessary to decompose a large design problem into several smaller design problems that can be solved consistently and efficiently. Multilevel design strategies, such as analytical target cascading, deserve further study for these large-scale manufacturing applications.

APPENDICES

APPENDIX A

Abbreviations

AIO	all-in-one problem
ATC	analytical target cascading
CAVA	compliant assembly variation analysis
DIRECT	DIvided RECTangles optimization algorithm
FEA	finite element analysis
KCCs	key control characteristics
KPCs	key product characteristics
MIGA	multi-island genetic algorithm
MPC	multipoint constraint
NCGA	neighborhood cultivation genetic algorithm
PLP	principal locating point
SQP	sequential quadratic programming
s.t.	subject to

APPENDIX B

Nomenclature

$\mathbf{A}$	state transition matrix in a variation propagation model
b	budget requirement
b_g	global budget requirement
b_l	local budget requirement
$\mathbf{B}$	input matrix in a variation propagation model
$\mathbf{C}$	matrix in a rigid variation propagation model
c	system cost including all costs of stations and incoming parts
c_{ij}	cost of a station j at level i
c_τ	cost with respect to process tolerance
c_t	cost with respect to incoming part
$\mathbf{D}$	matrix related to a variation propagation model
$f(\mathbf{x})$	objective function to be minimized with respect to $\mathbf{x}$
$\mathbf{f}(\mathbf{x})$	vector of objective functions to be minimized with respect to $\mathbf{x}$
$f_i(\mathbf{x})$	ith objective function to be minimized with respect to $\mathbf{x}$
$\mathbf{F}_t$	tolerance transfer model
F_c	cost-tolerance model
$\mathbf{g}(\mathbf{x})$	vector of inequality constraint functions

$\mathbf{h}(\mathbf{x})$	vector of equality constraint functions
$(\cdot)^H$	quantities computed at a higher level in the hierarchy
i	level in the hierarchy
j	station at ith level in the hierarchy
k	station index in a sequence or a vector
k_1	fixed cost in a cost-tolerance model
k_2	cost of producing a single component dimension to a specified tolerance t in a cost-tolerance model
k_3	sensitivity of the process cost to changes in tolerance specifications
$(\cdot)^L$	quantities computed at a lower level in the hierarchy
$\mathbf{M}$	relocation matrix in a variation propagation model
M_{ij}	number of "child" stations
N	number of levels in the hierarchy; number of NC blocks
NC_k	kth NC block
n_{PLP}	total number of principal locating points
$\mathbf{p}$	position design vector
P	principal locating point
P_{4way}	four-way pin-hole locating pair
P_{2way}	two-way pin-hole locating pair
$\mathbf{P}$	deformation matrix in a variation propagation model
q	assembly quality
q_c	critical quality requirement
q_s	assembly quality requirement
S	sensitivity
S_{max}	maximum sensitivity

$\mathbf{S}$	sensitivity matrix in a variation propagation model
$\mathbf{S}_{\mathbf{A}_k}$	sensitivity matrix obtained using CAVA
$\mathbf{S}_{\mathbf{B}_k}$	sensitivity matrix obtained using CAVA
SI	sensitivity index
t	tolerance variable
$\mathbf{t}$	product tolerance vector
$\mathbf{t}_{ij}$	product tolerance vector, output of level i, station j
$\boldsymbol{\tau}$	process tolerance vector
$\boldsymbol{\tau}_{ij}$	process tolerance vector at i, station j
$(\cdot)^T$	transpose
U_i	upper bound in ε-constraint method
$\mathbf{u}$	fixture (tooling) deviation vector
$\mathbf{v}$	additive sensor noise vector
$\mathbf{w}$	noise vector
$\mathbf{x}$	vector of design variables, a point in $\Re^n$; product deviation vector
X	coordinate of a locator
$\mathbf{y}$	part variation vector
Y	coordinate of a locator
$\mathbf{z}$	group of design variables
Z	coordinate of a locator
λ	eigenvalue
λ_{max}	maximum eigenvalue

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